10

Direct Proportion

By studying this lesson you will be able to;

- identify direct proportions,
- solve problems related to direct proportion using the unitary method,
- solve problems related to direct proportion using the definition,
- write the relationship between two directly proportional quantities in the form $y = kx$,
- solve problems related to the conversion of foreign currencies using the knowledge on direct proportions.

10.1 Introduction to direct proportion

The way the price of a certain type of pen varies depending on the quantity of pens is given in the following table.

Number of pens	Price (Rs)
1	15
2	30
3	45
4	60
5	75
6	90

It is clear from the above table that the price increases as the number of pens increases. Let us consider the number of pens and the price as two quantities.

Based on the above example, a few ratios of different amount of pens and the ratios of the corresponding prices are shown in the following table. Observe that these ratios are equal.

Ratio of two amounts of pens	Ratio of the corresponding prices
1 : 2	15 : 30 = 1 : 2
1 : 3	15 : 45 = 1 : 3
2 : 3	30 : 45 = 2 : 3
3 : 5	45 : 75 = 3 : 5
2 : 5	30 : 75 = 2 : 5

Two distinct quantities are said to be in direct proportion if they increase or decrease in the same ratio.

Therefore, if two quantities are in direct proportion, then when one quantity increases, the other quantity will also increase in the same ratio.
Similarly, if two quantities are in direct proportion and one quantity decreases, then the other quantity will also decrease in the same ratio.

Exercise 10.1

1. For each of the cases given below, write whether the two quantities are directly proportional or not.

- **a.** The number of books and the price
- **b.** The distance travelled by an object moving at a constant speed and the time taken for the journey.
- **c.** The speed of a vehicle and the time taken to travel a certain distance
- **d.** The length of a side of a square and its perimeter
- **e.** The length of a side of a square and its area
- **f.** The number of people needed to finish a task and the number of days taken for it
- **g.** The number of units of electricity consumed by a household and the monthly bill

10. 2 Solving problems related to direct proportion using the unitary method

Suppose we want to find the price of 5 cakes of a certain type of soap, given that the price of 3 cakes of soap of that type is Rs 120.

As you have learnt in previous grades, we can first find the price of one cake of soap and thereby easily find the price of 5 cakes of soap.

Price of 3 cakes of soap = Rs 120
Price of 1 cake of soap = Rs 120 ÷ 3
= Rs 40
Price of 5 cakes of soap = Rs 40 × 5
= Rs 200

This method of calculation can also be explained as follows.

There are two quantities. They are the number of cakes of soap and the price. Initially the price of one cake of soap is found. It is Rs. 40. To find the price of five cakes of soap, the price of one cake of soap is multiplied by 5. Here the price of one cake of soap is clearly the constant value of the following fraction.

$$\frac{\text{price of 3 cakes of soap}}{\text{number of cakes of soap}}$$

The method of solving a problem based on the value of a unit is called the unitary method.

Let us learn how to solve problems related to direct proportion using the unitary method by considering a few examples.

Example 1

If a person walking at a constant speed takes 5 minutes to walk 800 m, calculate the distance he walks in 12 minutes.

Distance walked in 5 minutes in metres = 800
Distance walked in 1 minute in metres = 800 ÷ 5
= 160
Distance walked in 12 minutes in metres = 160 × 12
= 1 920

∴ The distance walked in 12 minutes is 1920 m.

Example 2

If the mass of 10 identical balls used in a cricket match is 3 kg, what is the mass of 3 such balls?

Mass of 10 balls in kilogrammes = 3
Mass of 1 ball in grammes = $3000 \div 10$
= 300
Mass of 3 balls in grammes = 300×3
= 900

$\therefore$ The mass of 3 balls is 900g.

Do the following exercise using the unitary method.

Exercise 10.2

1. If the price of 8 oranges is Rs 320, find the price of 5 such oranges.

2. If the price of 5 m of a certain fabric is Rs 750, find the price of 12 m of that fabric.

3. If the mass of a parcel containing 15 apples is 3.6 kg, find the mass of a parcel containing 8 such apples.

4. If a printing machine makes 240 copies in 5 minutes, determine the number of copies it makes in 12 minutes.

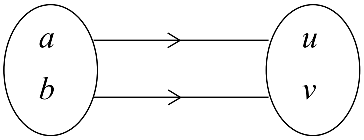
5. If a motor vehicle moving at a constant speed travels 12 km in 15 minutes, calculate the distance it travels in 40 minutes.

6. If a motorbike can travel 90 km on 2 *l* of petrol, find the distance it can travel on 5 *l* of petrol.

7. If the time taken for a tank of capacity 1000 litres to be filled using a pump that releases water at a constant rate is 5 minutes, find the time taken in seconds to fill a tank of capacity 1600 litres.

10.3 Solving problems related to direct proportion using the definition

In the first section of this lesson it was explained that if two quantities are directly proportional, then the ratio of any two values of the first quantity is equal to the ratio of the corresponding values of the second quantity. This can be shown algebraically as below. Let us assume that the price of an amount a of a certain item is Rs u and the price of an amount b of the same item is Rs v.



Then we can write $a : b = u : v$.

This can be expressed in terms of fractions as $\frac{a}{b} = \frac{u}{v}$ (or $\frac{b}{a} = \frac{v}{u}$).

Let us learn how to solve problems related to direct proportion using this feature by considering the following examples.

Example 1

If the price of 5 mangoes is Rs 75, find the price of 8 mangoes.

Let us take the price of 8 mangoes as x. Then we can illustrate this information using an arrow diagram as shown below.

$$\begin{tikzcd}
\text{Number of mangoes} & \text{Price (Rs)} \\
5 \arrow[r] & 75 \\
8 \arrow[r] & x
\end{tikzcd}$$

Using this as the base, let us write an algebraic equation as shown below and by solving it, find the value of x; that is, the price of 8 mangoes.

$$5 : 8 = 75 : x$$

Therefore, $$\frac{5}{8} = \frac{75}{x}$$

$$5x = 75 \times 8$$

$$x = \frac{75 \times 8}{5}$$

$$x = 120$$

Accordingly, the price of 8 mangoes is Rs 120.

Example 2

Find the price at which an item bought for Rs 500 should be sold to earn a profit of 15%.

Let us write the information in this problem as follows, so that we can use direct proportions. “If the selling price of an item bought for Rs 100 is Rs 115 (since the profit is 15%), find the selling price of an item bought for Rs 500.”

Let us assume that the selling price of an item bought for Rs 500 is Rs x.

$$\begin{array}{ccc} \text{Purchase price (Rs)} & & \text{Selling price (Rs)} \\ 100 & \longrightarrow & 115 \\ 500 & \longrightarrow & x \end{array}$$

$100 : 500 = 115 : x$

$$\frac{100}{500} = \frac{115}{x}$$

$$100\,x = 115 \times 500$$

$$x = \frac{115 \times 500}{100}$$

$$x = 575$$

Accordingly, the selling price should be Rs 575.

Exercise 10.3

1. For each of the proportions given below, write the suitable value in the blank space.

a. 2 : 5 = 8 : ......
b. 3 : 4 = ... : 20
c. 5 : 3 = 40 : ......
d. 4 : 1 = ... : 8
e. 8 : ...... = 24 : 15
f. ...... : 6 = 35 : 30

2. Solve each problem given below using proportions, by first drawing an arrow diagram and then writing an algebraic equation.

a. If the price of 10 kg of rice is Rs 850, find the price of 7 kg of rice.

b. If the mass of 9 cm^3 of a certain type of metal is 108 g, find the mass of 12 cm^3 of this metal.

c. If the distance travelled in 4 hours by a motorbike moving at a constant speed is 240 km, find the distance travelled by it in 3 hours.

d. Find the amount needed to buy an item worth Rs 800 from a shop which offers a discount of 3%.

e. If a commission of 12% is given when an item is sold, what is the commission given for an item worth Rs 15 000?

f. If the price of 4 pencils is Rs 48, find the number of pencils that can be bought for Rs 132.

g. If the price of 12 bottles is Rs 4800, find the number of bottles that can be bought for Rs 6000.

10.4 Solving problems related to direct proportion algebraically

If the price of 1 pen is Rs 15, then

- the price of 2 pens is Rs 30.
- the price of 3 pens is Rs 45.
- the price of 4 pens is Rs 60.

If we consider the above four instances, it can be observed that if the amount of money spent is divided by the number of pens, the value that is obtained is a constant.

That is, $\dfrac{\text{money spent}}{\text{number of pens}} = \text{constant value.}$

This constant value is the price of one pen. Accordingly, if the money spent for x pens is y,

we can write $\dfrac{y}{x} = k$; here k is a constant.

This equation can also be written as $y = kx$.

Let us learn how to solve problems related to direct proportions using the above algebraic equation, by considering the following examples.

Example 1

If the price of 3 exercise books is Rs 75, find the price of 5 such exercise books.
Let us take the number of books as x and the price as y.

Then we can write $y = kx$; where k is a constant. The value of k can be found using the information given in the problem.

Since the price of 3 exercise books is Rs 75, when $x = 3$, $y = 75$.

By substituting these values in the equation we obtain, $75 = k \times 3$.
By solving this we obtain $k = 25$.

By substituting this value of k in the first equation, we obtain the relationship between x and y as $y = 25x$.

Now, using this equation, for any value of x the corresponding value of y and for any value of y the corresponding value of x can be found.
In this problem, since we need the price of 5 exercise books, y needs to be found when $x = 5$.

By substituting $x = 5$ in the equation $y = 25x$ we get,

$$\begin{aligned} y &= 25 \times 5 \\ &= 125 \end{aligned}$$

Accordingly, the price of 5 exercise books is Rs 125.

Example 2

If a vendor sells an item he bought for Rs 500 such that he earns a profit of 20%, determine the selling price of the item.
Taking the purchase price of the item as x and selling price as y we can write $\frac{y}{x} = k$.

Since the selling price is Rs 120 when the purchase price is Rs 100, we obtain $\frac{120}{100} = k$.
Let us assume that the selling price of an item bought for Rs 500 is y. Then we obtain the equation $\frac{y}{500} = k$.

Since k is a constant, we can write, $\frac{y}{500} = \frac{120}{100}$.

Therefore, $y = \frac{120 \times 500}{100}$.

$y = 600.$

$\therefore$ The selling price of the item is Rs 600.

Exercise 10.4

Do the problems in this exercise, using the algebraic equation method.

1. If the price of 3 shirts is Rs 1200, find the price of 5 shirts.

2. If the daily wage of 8 labourers who are paid equal wages is Rs 7200, find the daily wage of 3 labourers.

3. If a distance of 25 m is represented by 5 cm on a map drawn to scale, find the actual distance represented by 8 cm on this map.

4. If a machine in a factory produces 7500 drink bottles in 5 hours, find the number of drink bottles it produces in 7 hours.

5. A bookstore offers a discount of 8% on every book that is purchased. Find the amount a person has to pay if he purchases books worth Rs 1 200.

10.5 Foreign currency

We know that every country has its own currency unit and that the rate of conversion of the currency of one country to that of another country varies depending on the countries. The rate at which one country exchanges its currency with that of another country is called the **exchange rate**. This rate is not a constant value; it increases and decreases daily due to various reasons.

The currency units used by certain countries and their exchange rates with respect to the Sri Lankan rupee on a particular day, is given below.

Here the exchange rate given is the value of one foreign currency unit in Sri Lankan rupees.

Country/Union	Foreign currency unit	Exchange rate (Rs)
United States of America	American Dollar	151.20
England	Sterling Pound	185.90
European Union	Euro	160.60
Japan	Yen	1.33
India	Indian Rupee	2.26
Saudi Arabia	Saudi Riyal	40.32
Singapore	Singapore Dollar	107.30

(From the internet on 2017-03-05)

Now let us consider how to solve problems related to exchange rates using proportions.

Example 1

On a day that the exchange rate is Rs 151 for an American dollar, how many Sri Lankan rupees will a person who converts 50 American dollars receive?

$$\begin{aligned}\text{Value of 1 American dollar} &= \text{Rs } 151\\ \text{Value of 50 American dollars} &= \text{Rs } 151 \times 50\\ &= \text{Rs } \underline{\underline{7550}}\end{aligned}$$

Therefore, the person will receive Rs 7550.

Example 2

A person visiting England, converted Rs 74 000 into sterling pounds on a day when the exchange rate was Rs 185 for a sterling pound. How many sterling pounds did he receive?

$$\begin{aligned}\text{The value of 185 Sri Lankan rupees} &= 1 \text{ sterling pound}\\ \text{The value of 1 Sri Lankan rupee} &= \frac{1}{185} \text{ sterling pounds}\\ \text{The value of 74 000 Sri Lankan rupees} &= \frac{1}{185} \times 74\,000 \text{ sterling pounds}\\ &= 400 \text{ sterling pounds}\end{aligned}$$

(It is easy to simplify this if we keep $\frac{1}{185}$ as a fraction without converting it into a decimal number). Therefore, the amount of sterling pounds he received is 400.

Exercise 10.5

Do the following exercise by using the exchange rate table given earlier.

1. If the monthly salary of a person working in a foreign country is 1500 American dollars, what is his salary in Sri Lankan rupees?

2. If the price of a television set imported from Japan is 12 500 yen, what is its value in Sri Lankan rupees?

3. A monthly allowance of 2500 sterling pounds is given to a scholarship student engaged in further studies in Great Britain. How much is this amount in Sri Lankan rupees?

4. A sports equipment in a duty free shop is worth 750 euros. How many Sri Lankan rupees have to be paid to purchase it?

5. A pilgrim who travels to India, converts 56 000 Sri Lankan rupees into Indian rupees. How many Indian rupees does he receive?

6. How many Singapore dollars are received when readymade garments worth Rs 600 880 are exported from Sri Lanka to Singapore?

11

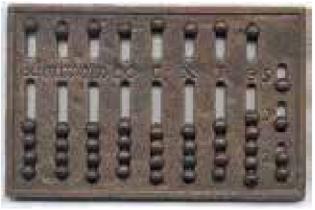
The Calculator

By studying this lesson you will be able to,

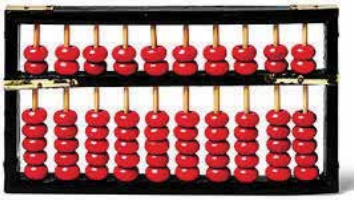
- identify and use the keys of =, %, x^2 and $\sqrt{x}$ in the scientific calculator.

The Calculator

Since ancient times, humans have used various devices to perform calculations. During the period when animal husbandry commenced, men used pebbles to count the number of animals they owned. There is evidence to show that counting was done by drawing lines on clay tablets during a later period. The Egyptians started using a device known as the abacus for calculations in 1000 B.C. The modern day abacus was invented by the Chinese in the 15th century. John Napier who lived in the 17th century invented "Napier's bones" which is a device that was used to calculate products and quotients of numbers.



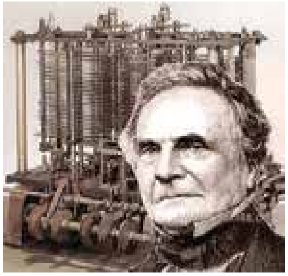
Egyptian abacus



Modern abacus

The first mechanical calculator was invented by the French mathematician Blaise Pascal(1623-1662). In the year 1883, the Englishman Charles Babbage (1791-1871) introduced a more advanced calculator. Based on this, the electrically operated computer was invented. The production of the modern compact calculators commenced with the development of electronics.

Blaise Pascal



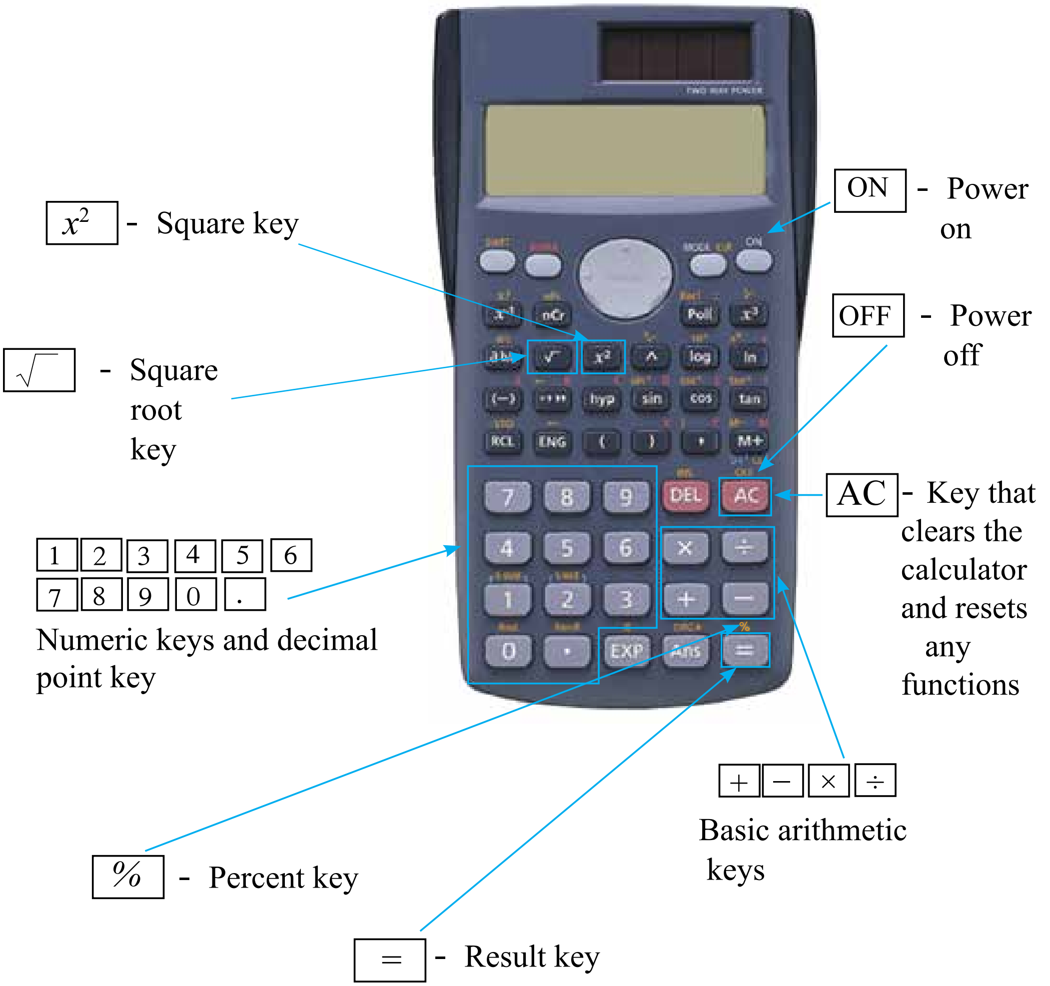
Charles Babbage

Nowadays, two types of calculators are manufactured. They are the ordinary calculators and the scientific calculators. Ordinary calculators can perform only normal mathematical operations such as addition, subtraction, multiplication, division and square roots. However, by using a scientific calculator, operations such as x^2, x^3 $\sqrt[x]{y}$, 10^x can be performed.

The Scientific Calculator

The scientific calculator, like the ordinary calculator, consists of a key pad to enter data and a screen to display results. However, the number of keys on the keypad and the number of digits that can be displayed on the screen of a scientific calculator are greater than those of an ordinary calculator.

Let us identify the keys on the keypad of a scientific calculator.



11.1 Performing calculations using a calculator

When performing calculations using a calculator, the keys need to be pressed in a specific order.

Example 1

The order in which the keys need to be pressed to obtain the value of 27 + 35 is the following.

$\boxed{\text{ON}} \rightarrow \boxed{2} \rightarrow \boxed{7} \rightarrow \boxed{+} \rightarrow \boxed{3} \rightarrow \boxed{5} \rightarrow \boxed{=} \ \boxed{62}$

Example 2

The order in which the keys need to be pressed to obtain the value of 208 − 159 is the following.

$\boxed{\text{ON}} \rightarrow \boxed{2} \rightarrow \boxed{0} \rightarrow \boxed{8} \rightarrow \boxed{-} \rightarrow \boxed{1} \rightarrow \boxed{5} \rightarrow \boxed{9} \rightarrow \boxed{=} \ \boxed{49}$

Example 3

The order in which the keys need to be pressed to obtain the value of 5.25×35.4 is the following.

$\boxed{\text{ON}} \rightarrow \boxed{5} \rightarrow \boxed{.} \rightarrow \boxed{2} \rightarrow \boxed{5} \rightarrow \boxed{\times} \rightarrow \boxed{3} \rightarrow \boxed{5} \rightarrow \boxed{.} \rightarrow \boxed{4} \rightarrow \boxed{=} \ \boxed{185.85}$

Example 4

The order in which the keys need to be pressed to obtain the value of 5.52 ÷ 6 is the following.

$\boxed{\text{ON}} \rightarrow \boxed{5} \rightarrow \boxed{.} \rightarrow \boxed{5} \rightarrow \boxed{2} \rightarrow \boxed{\div} \rightarrow \boxed{6} \rightarrow \boxed{=} \ \boxed{0.92}$

To switch the calculator off after a calculation is done, press the $\boxed{\text{OFF}}$ key. If you wish to start another calculation without switching the calculator off, use the $\boxed{AC}$ key to clear the screen and reset the functions.

Example 5

Write the order in which the keys need to be pressed to do the following simplifications.

(i) 53 + 42 − 25 (ii) 35 × 45 ÷ 21

$\boxed{\text{ON}} \rightarrow \boxed{5} \rightarrow \boxed{3} \rightarrow \boxed{+} \rightarrow \boxed{4} \rightarrow \boxed{2} \rightarrow \boxed{-} \rightarrow \boxed{2} \rightarrow \boxed{5} \rightarrow \boxed{=} \ \boxed{70}$

$\boxed{\text{AC}} \rightarrow \boxed{3} \rightarrow \boxed{5} \rightarrow \boxed{\times} \rightarrow \boxed{4} \rightarrow \boxed{5} \rightarrow \boxed{\div} \rightarrow \boxed{2} \rightarrow \boxed{1} \rightarrow \boxed{=} \ \boxed{75}$

Exercise 11.1

Simplify each of the following using a calculator. Indicate the order in which the keys need to be pressed to obtain the correct answer.

a. 45 + 205
b. 350 − 74
c. 824 × 95
d. 3780 ÷ 35
e. 3.52 + 27.7
f. 43.5 − 1.45
g. 7.35 × 6.2
h. 134.784 ÷ 31.2
i. 12.5 ÷ 50 × 4.63
j. 15.84 − 6.75 × 3.52
k. 120.82 ÷ 0.0021 × 5
l. 0.006 ÷ 0.33 × 0.12

Performing calculations using an ordinary calculator or a scientific calculator

Let us consider how simplifications are done using a calculator when more than one operation is involved.

In simplifying 75 + 6 ÷ 3 using an ordinary calculator, when the keys are pressed in the order

$\boxed{\text{ON}} \to \boxed{7} \to \boxed{5} \to \boxed{+} \to \boxed{6} \to \boxed{\div} \to \boxed{3} \to \boxed{=}$, the operations are performed in the order that they have been entered and 27 is obtained as the answer. That is, a wrong answer is obtained by 75 + 6 ÷ 3 = 81 ÷ 3 = 27.
(This is wrong according to the “BODMAS Rule”.)

When we enter the above data in a scientific calculator in the same order, the operations are performed according to the accepted order of performing mathematical operations and the answer 77 is obtain by performing the operations as follows: 75 + 6 ÷ 3 = 75 + 2 = 77.

Note: When performing calculations using an ordinary calculator, the order in which data is entered should be chosen with care. However, the correct answer can be obtained when data is entered into a scientific calculator in the order in which they appear. Most manufacturers of calculators follow the BODMAS rule when programming their products. However, there are some calculators that do not perform the operations according to this rule. The order in which data should be entered into these calculators is indicated in the instruction booklet that is provided. If such a booklet is not available, an understanding of how the calculator operates can be gained by performing a few simple calculations. Otherwise, the operations that need to be performed initially have to be enclosed within brackets. For example, if the data in the expression 1 − 5 + 12 ÷ 3 × 2 is entered in the given order, some calculators may perform the multiplication before the division. However, according to the BODMAS rule, since multiplication and division have the same priority, moving from left to right, the division should be peformed first.

11.2 Using the $\boxed{\%}$ key in the scientific calculator

The % key is used to calculate percentages. Both the symbols "=" and "%" appear on the same key in most calculators. To activate the % key, press the $\boxed{\text{Shift}}$ key and then the $\boxed{=}$ key.

Example 1

The keys of the calculator need to be pressed in the following order to find 25% of 480.

$$\boxed{\text{ON}} \text{ or } \boxed{\text{AC}} \to \boxed{4} \to \boxed{8} \to \boxed{0} \to \boxed{\times} \to \boxed{2} \to \boxed{5} \to \boxed{\text{SHIFT}} \to \overset{\%}{\boxed{=}} \to \boxed{=} \quad \boxed{120}$$

Example 2

Let us express $\frac{2}{8}$ as a percentage. To do this, the keys of the calculator need to be pressed in the following order.

$$\boxed{\text{ON}} \to \boxed{2} \to \boxed{\div} \to \boxed{8} \to \boxed{\text{SHIFT}} \to \overset{\%}{\boxed{=}} \to \boxed{=} \quad \boxed{25}$$

Example 3

The keys of the calculator need to be pressed in the following order to find 35% of Rs 2500.

$$\boxed{\text{ON}} \to \boxed{2} \to \boxed{5} \to \boxed{0} \to \boxed{0} \to \boxed{\times} \to \boxed{3} \to \boxed{5} \to \boxed{\text{SHIFT}} \to \overset{\%}{\boxed{=}} \to \boxed{=} \quad \boxed{875}$$

Example 4

The population of a certain village is 550. Of this number, 66 are school children. To find what percentage of the population are school children, the keys of the calculator need to be pressed in the following order.

$$\boxed{\text{ON}} \to \boxed{6} \to \boxed{6} \to \boxed{\div} \to \boxed{5} \to \boxed{5} \to \boxed{0} \to \boxed{\text{SHIFT}} \to \overset{\%}{\boxed{=}} \to \boxed{=} \quad \boxed{12}$$

Exercise 11.2

1. Simplify each of the following using a calculator. Indicate the order in which the keys of the calculator need to be pressed to get the correct answer.

a. $350 \times 3\%$ **b.** $7520 \times 60\%$ **c.** $75.3 \times 5\%$

2. Using a calculator, express each of the following as a percentage.

a. $\frac{1}{5}$ **b.** $\frac{12}{25}$ **c.** $\frac{7}{20}$

Use a calculator to find the solution to each of the following problems.

3. A chair which was produced at a cost of Rs 450 was sold at a profit of 22%. What was the profit?

4. Of the 750 pupils in a certain school, 20% travel to school by bus. How many pupils travel to school by bus?

5. Nimal's monthly salary is Rs 35 000. Of this amount, he deposits Rs 7000 in a savings account. What percentage of his salary does he save?

6. Of the 650 students in a certain school, 143 learn music. Express the number of students who learn music as a percentage of the total number of students in the school.

7. It was stated that the amount of unfilled grain in a stock of paddy is less than 2%. There were 6 kg of unfilled grain in 350 kg of that stock. Was the statement that was made true?

11.3 Performing calculations using the $\boxed{x^2}$ key

To find the value of powers with index two such as 2^2, 5^2 and 3.21^2, we use the $\boxed{x^2}$ key.

Example 1

The order in which the keys need to be pressed to obtain the value of 3^2 is the following.

$\boxed{\text{ON}} \rightarrow \boxed{3} \rightarrow \boxed{x^2} \rightarrow \boxed{=} \rightarrow \boxed{9}$

Example 2

The order in which the keys need to be pressed to obtain the value of 4.1^2 is the following.

$\boxed{\text{AC}} \rightarrow \boxed{4} \rightarrow \boxed{\cdot} \rightarrow \boxed{1} \rightarrow \boxed{x^2} \rightarrow \boxed{=} \quad \boxed{16.81}$

Example 3

The order in which the keys need to be pressed to obtain the value of $5^2 \times 12^2$ is the following.

$\boxed{\text{AC}} \rightarrow \boxed{5} \rightarrow \boxed{x^2} \rightarrow \boxed{\times} \rightarrow \boxed{1} \rightarrow \boxed{2} \rightarrow \boxed{x^2} \rightarrow \boxed{=} \boxed{3600}$

Example 4

The order in which the keys need to be pressed to find the area of a square of side length 6 cm is the following.
Since the area of the square $= 6 \times 6 \text{ cm}^2$

$\boxed{\text{ON}} \rightarrow \boxed{6} \rightarrow \boxed{x^2} \rightarrow \boxed{=} \rightarrow \boxed{36}$

$\therefore$ The area of the square is 36 cm².

Exercise 11.3

1. Find the value of each of the following powers using a scientific calculator. Indicate the order in which the keys of the calculator need to be pressed to obtain the correct answer.

a. 2^2 **b.** 8^2 **c.** 127^2
d. 3532^2 **e.** 3.5^2 **f.** 6.03^2

2. Find the value of each of the following using a scientific calculator. Indicate the order in which the keys of the calculator need to be pressed to obtain the correct answer.

a. 3×5^2 **b.** $3^2 \times 4^2$ **c.** $(3.5)^2$
d. $4^2 + 3^2$ **e.** $10^2 - 6^2$ **f.** $10^2 - 3^2 \times 5$

11.4 Performing calculations using the $\boxed{\sqrt{}}$ key in a scientific calculator

To find the value of the square root of a number we use the $\boxed{\sqrt{}}$ key.

Example 1

The order in which the keys of a scientific calculator need to be pressed to find the value of $\sqrt{25}$ is the following.

$\boxed{\text{ON}} \rightarrow \boxed{\sqrt{}} \rightarrow \boxed{2} \rightarrow \boxed{5} \rightarrow \boxed{=} \rightarrow \boxed{5}$

Example 2

The order in which the keys of a scientific calculator need to be pressed to find the value of $\sqrt{44\,521}$ is the following.

$\boxed{\text{ON}} \rightarrow \boxed{\sqrt{}} \rightarrow \boxed{4} \rightarrow \boxed{4} \rightarrow \boxed{5} \rightarrow \boxed{2} \rightarrow \boxed{1} \rightarrow \boxed{=} \rightarrow \boxed{211}$

Example 3

The order in which the keys of a scientific calculator need to be pressed to find the value of $\sqrt{5.29}$ is the following.

$\boxed{\text{ON}} \rightarrow \boxed{\sqrt{\ }} \rightarrow \boxed{5} \rightarrow \boxed{.} \rightarrow \boxed{2} \rightarrow \boxed{9} \rightarrow \boxed{=}$ 2.3

Exercise 11.4

1. Find the square root of each of the following numbers using a scientific calculator. Indicate the order in which the keys of the calculator need to be pressed to obtain the correct answer.

a. 64 **b.** 81 **c.** 2704
d. 3356 **e.** 3500 **f.** 362404

2. Find the value of each of the following using a scientific calculator. Indicate the order in which the keys of the calculator need to be pressed to obtain the correct answer.

a. $\sqrt{49}$ **b.** $\sqrt{121}$ **c.** $\sqrt{625}$
d. $\sqrt{20.25}$ **e.** $\sqrt{5.76}$ **f.** $\sqrt{0.1225}$

For further knowledge

The order in which the keys of a calculator need to be pressed to find the value of $\sqrt{4^2 + 3^2}$ is the following.

$\boxed{\text{ON}} \rightarrow \boxed{\sqrt{\ }} \rightarrow \boxed{(} \rightarrow \boxed{4} \rightarrow \boxed{x^2} \rightarrow \boxed{+} \rightarrow \boxed{3} \rightarrow \boxed{x^2} \rightarrow \boxed{)} \rightarrow \boxed{=}$ 5

Miscellaneous Exercise

1. Simplify using a scientific calculator. Indicate the order in which the keys need to be pressed to obtain the correct answer.

a. $5 + 6 \div 2 + 4 \times 5$ **b.** $6225 + 37 \times 0.25$ **c.** $42.48 \div 5.31$
d. $428 + 627 \times 5\ \%$ **e.** $5.3^2 \div 6.01$ **f.** $\frac{7}{130} \times 2\ \% + 560$

2. Of the 35 seeds that Saman planted, 21 germinated. Using a scientific calculator, find what percentage of the seeds that were planted germinated.

3. Nimal received a salary increment of 12%. If his salary before the increment was Rs 45 200, how much was his salary after the increment?

4. Find the value of a if $a = 1.33^2$.

5. Find the value of p if $p = \sqrt{18.49 - 2}$.

12

Indices

By studying this lesson you will be able to;

- identify the laws of indices on the product of powers, the quotient of powers and the power of a power,
- simplify algebraic expressions using the above mentioned laws of indices,
- identify the zero index and negative indices and simplify algebraic expressions containing these.

Indices

You have learnt about powers of numbers such as 2^1, 2^2 and 2^3 in previous grades. The values of these powers can be obtained as follows.

$$\begin{aligned} 2^1 &= 2 \\ 2^2 &= 2 \times 2 = 4 \\ 2^3 &= 2 \times 2 \times 2 = 8 \\ \vdots & \qquad \vdots \end{aligned}$$

You have also learnt about powers of algebraic symbols such as x^1, x^2 and x^3 .These can be expanded and written as follows.

$$\begin{aligned} x^1 &= x \\ x^2 &= x \times x \\ x^3 &= x \times x \times x \\ \vdots & \qquad \vdots \end{aligned}$$

Moreover, you have learnt how to write the expanded form of a product of powers of algebraic symbols and numerical values. For example, $5^2a^3b^2$ is written in expanded form as follows.

$$5^2a^3b^2 = 5 \times 5 \times a \times a \times a \times b \times b.$$

You have also learnt that a power of a product such as $(xy)^2$ can be expressed as a product of powers as x^2y^2 and a power of a quotient such as $\left(\frac{x}{y}\right)^2$ can be expressed as a quotient of powers as $\frac{x^2}{y^2}$.

Do the following review exercise to recall what you have learnt in previous grades regarding indices.

Review Exercise

1. Evaluate the following.

i. 2^5 **ii.** $(-3)^2$ **iii.** $(-4)^2$

iv. $\left(\frac{2}{3}\right)^2$ **v.** $(-3)^3$ **vi.** $(-4)^3$

2. Fill in the blanks.

i.
$$\begin{aligned}(xy)^2 &= (xy) \times \ldots\ldots \\ &= \ldots\ldots \times \ldots\ldots \times x \times y \\ &= x \times x \times \ldots\ldots \times \ldots\ldots \\ &= \underline{\underline{x^2 \times y^2}}\end{aligned}$$

ii.
$$\begin{aligned}(pq)^3 &= \ldots\ldots \times \ldots\ldots \times \ldots\ldots \\ &= p \times q \times \ldots\ldots \times \ldots\ldots \\ &= p \times p \times p \times \ldots\ldots \times \ldots\ldots \times \ldots\ldots \\ &= \underline{\underline{p^3 \times q^3}}\end{aligned}$$

iii.
$$\begin{aligned}(2ab)^2 &= \ldots\ldots \times \ldots\ldots \\ &= \ldots\ldots \times \ldots\ldots \times b \times \ldots\ldots \times \ldots\ldots \times b \\ &= 2 \times 2 \times \ldots\ldots \times \ldots\ldots \times \ldots\ldots \times \ldots\ldots \\ &= \underline{\underline{4a^2b^2}}\end{aligned}$$

iv.
$$\begin{aligned}9p^2q^2 &= \ldots\ldots^2 \times p^2 \times q^2 \\ &= \ldots\ldots \times \ldots\ldots \times p \times p \times \ldots\ldots \times \ldots\ldots \\ &= (3 \times p \times q) \times (\ldots\ldots \times \ldots\ldots \times \ldots\ldots) \\ &= \underline{\underline{(3pq)^2}}\end{aligned}$$

3. Expand and write each of the following expressions as a product.

i. $2a^2$ **ii.** $3x^2y^2$ **iii.** $-5p^2q$

iv. $(-3)^5$ **v.** $(ab)^3$ **vi.** $x^4 \times y^4$

12.1 Products of powers with the same base

2^3 and 2^5 are two powers with the same base.
They can be expanded and written as follows.

$$2^3 = 2 \times 2 \times 2 \text{ and}$$
$$2^5 = 2 \times 2 \times 2 \times 2 \times 2$$

Let us obtain the product of these two powers.

$$\begin{aligned}2^3 \times 2^5 &= (2 \times 2 \times 2) \times (2 \times 2 \times 2 \times 2 \times 2) \\ &= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \\ &= 2^8\end{aligned}$$

2 is repeatedly multiplied three times in 2^3. Furthermore, 2 is repeatedly multiplied five times in 2^5.

Therefore, when multiplying 2^3 by 2^5, 2 is repeatedly multiplied $3 + 5 = 8$ times.

This can be expressed as $2^3 \times 2^5 = 2^{3+5} = 2^8$.

It is important to remember that when two powers are multiplied, their indices can be added only if they have the same base. Then the base of the power that is obtained is also this common base.

Let us obtain the product $x^3 \times x^5$ accordingly. Since x^3 and x^5 have the same base, the two indices are added to get the product.
That is, $x^3 \times x^5 = x^{3+5}$
$= x^8$
This can be expressed as a law of indices as follows.

$$a^m \times a^n = a^{m+n}$$

This law can be extended to any number of powers.
For example,

$$a^m \times a^n \times a^p = a^{m+n+p}$$

Let us understand how this law is used to simplify expressions by considering some examples.

Example 1

Simplify the following.

i. $x^2 \times x^5 \times x$ ii. $a^2 \times b^2 \times a^2 \times b^3$ iii. $2x^2 \times 3x^5$

i.

$$x^2 \times x^5 \times x = x^{2+5+1} \text{ (since } x = x^1) $$
$$= \underline{\underline{x^8}}$$

ii.

$$\begin{aligned} a^2 \times b^2 \times a^2 \times b^3 &= a^2 \times a^2 \times b^2 \times b^3 \\ &= a^{2+2} \times b^{2+3} \\ &= a^4 \times b^5 \\ &= \underline{\underline{a^4 b^5}} \end{aligned}$$

iii.

$$\begin{aligned} 2x^2 \times 3x^5 &= 2 \times x^2 \times 3 \times x^5 \\ &= 2 \times 3 \times x^2 \times x^5 \\ &= 6x^{2+5} \\ &= \underline{\underline{6x^7}} \end{aligned}$$

Do the following exercise using the law of indices on the product of powers.

Exercise 12.1

1. Fill in the blanks.

i. $2^5 \times 2^2$

$2^5 \times 2^2 = 2^{\ldots + \ldots}$

$= \underline{\underline{2^{\ldots}}}$

ii. $x^4 \times x^2$

$x^4 \times x^2 = x^{\ldots + \ldots}$

$= \underline{\underline{x^{\ldots}}}$

iii. $a^3 \times a^4 \times a$

$a^3 \times a^4 \times a = a^{\ldots + \ldots + \ldots}$

$= \underline{\underline{a^{\ldots}}}$

iv. $5p^3 \times 3p$

$= 5 \times \ldots \times 3 \times \ldots$

$= 15p^{\ldots + \ldots}$

$= 15\ldots$

v. $x^2 \times y^3 \times x^5 \times y^5$

$= x^{\ldots} \times x^{\ldots} \times y^{\ldots} \times y^{\ldots}$

$= x^{\ldots + \ldots} \times y^{\ldots + \ldots}$

$= \ldots^{\ldots} \times \ldots^{\ldots}$

2. Join each expression in column A with the expression in column B which is equal to it.

A	B
$x^3 \times x^7$	x^7
$x^5 \times x^2 \times x$	x^8
$x^7 \times x$	x^9
$x^2 \times x^2 \times x^6$	x^{10}
$x^2 \times x^3 \times x^2 \times x$	

3. Simplify and find the value.

i. $3^5 \times 3^5$

ii. $7^2 \times 7^3 \times 7$

4. Simplify.

i. $x^3 \times x^6$

ii. $x^2 \times x^2 \times x^2$

iii. $a^3 \times a^2 \times a^4$

iv. $2x^3 \times x^5$

v. $5p^2 \times 2p^3$

vi. $4x^2 \times 2x \times 3x^5$

vii. $m^2 \times 2n^2 \times m \times n$

viii. $2a^2 \times 3b^2 \times 5a \times 2b^3$

5. A pair of positive integral values that m and n can take so that the equation $x^m \times x^n = x^8$ holds true is 3 and 5. Write all such pairs of positive integral values.

6. Write a value of a for which the equation $a^2 + a^3 = a^5$ holds true and a value of a for which it is false.

12.2 Quotients of powers with the same base

Let us see whether there is a law of indices for the quotients of powers with the same base, similar to the one obtained above for the product.

$x^5 \div x^2$ can also be expressed as $\frac{x^5}{x^2}$.

$$\text{Now, } \frac{x^5}{x^2} = \frac{x \times x \times x \times x \times x}{x \times x}$$

$$= x \times x \times x$$

$$= \underline{\underline{x^3}}$$

$\therefore \frac{x^5}{x^2} = x^3$. When the index of the power in the numerator is 5 and the index of the power in the denominator is 2, then the index of the quotient is 5 – 2 = 3. The base of the quotient is x, which is the common base of the original two powers.

Therefore, $x^5 \div x^2$ can be simplified easily by subtracting the indices as follows.

$$x^5 \div x^2 = x^{5-2} = x^3$$

When powers with the same base are divided, the index of the divisor is subtracted from the index of the dividend. The base remains the same.

$$\boxed{a^m \div a^n = a^{m-n}}$$

It is important to remember this law of indices too.
Let us understand how this law is used to simplify expressions by considering the following examples.

Example 1

Simplify the following expressions.

a. $x^5 \times x^2 \div x^3$

$$(x^5 \times x^2) \div x^3 = x^{5+2} \div x^3$$
$$= x^{7-3}$$
$$= \underline{\underline{x^4}}$$

b. $4x^8 \div 2x^2$

$$4x^8 \div 2x^2 = \frac{\overset{2}{\cancel{4}}x^8}{\underset{1}{\cancel{2}}x^2}$$
$$= 2x^{8-2}$$
$$= \underline{\underline{2x^6}}$$

c. $\frac{a^3 \times a^2}{a}$

$$\frac{a^3 \times a^2}{a} = a^{3+2-1}$$
$$= \underline{\underline{a^4}}$$

Now do the following exercise.

Exercise 12.2

1. Simplify using the laws of indices.

i. $a^5 \div a^3$ **ii.** $\dfrac{x^7}{x^2}$ **iii.** $2x^8 \div x^3$ **iv.** $4p^6 \div 2p^3$

v. $\dfrac{10m^5}{2m^2}$ **vi.** $\dfrac{x^2 \times x^4}{x^3}$ **vii.** $n^5 \div (n^2 \times n)$ **viii.** $\dfrac{2x^3 \times 2x}{4x}$

ix. $\dfrac{x^5 \times x^2 \times 2x^6}{x^7 \times x^2}$ **x.** $\dfrac{a^5 \times b^3}{a^2 \times b^2}$ **xi.** $\dfrac{2p^4 \times 2q^3}{p \times q}$

2. Write five pairs of positive integral values for m and n which satisfy the equation $a^m \div a^n = a^8$

3. For each of the algebraic expressions in column A, select the algebraic expression in column B which is equal to it and combine the two expressions using the " = " sign.

A	B
$2a^5 \div 2a^2$	a
$a^6 \div a^4$	a^2
$\dfrac{a^7 \times a^2}{a^6}$	a^3
$\dfrac{a^3}{a}$	
$\dfrac{4a^5 \times a}{4a^3}$	

12.3 Negative indices

In the previous section we identified that $x^5 \div x^2$ is x^3. We know that this can be obtained by expanding $\frac{x^5}{x^2}$ and simplifying it as follows.

$$\frac{\overset{1}{\cancel{x}} \times \overset{1}{\cancel{x}} \times x \times x \times x}{\cancel{x}_1 \times \cancel{x}_1} = x^3$$

Let us simplify $x^2 \div x^5$ in a similar manner.

i. When expanded,

$$\frac{x^2}{x^5} = \frac{\overset{1}{\cancel{x}} \times \overset{1}{\cancel{x}}}{\overset{1}{\cancel{x}} \times \overset{1}{\cancel{x}} \times x \times x \times x}$$

$$= \underline{\underline{\frac{1}{x^3}}}$$

ii. Using laws of indices,

$$\frac{x^2}{x^5} = x^{2-5}$$

$$= \underline{\underline{x^{-3}}}$$

The two simplifications of $x^2 \div x^5$ obtained in (i) and (ii) above must be equal. Therefore, $\frac{1}{x^3} = x^{-3}$. Observe here that the index of the power in the denominator changes signs when it is brought to the numerator. This is an important feature related to indices which can be used when we need to change a negative index into a positive index. We can similarly write $x^3 = \frac{1}{x^{-3}}$.

This law can be expressed as follows.

$$x^n = \frac{1}{x^{-n}}$$

Accordingly, $a^{-m} = \frac{1}{a^m}$, $a^m = \frac{1}{a^{-m}}$, $\frac{a^{-m}}{a^{-n}} = \frac{a^n}{a^m}$ (By applying the above feature to both powers simultaneously)

We can use this law of indices to simplify algebraic expressions as shown in the following examples.

Example 1

Evaluate the following.

(i) 2^{-5} (ii) $\frac{1}{5^{-2}}$

i. $2^{-5} = \frac{1}{2^5}$

$$= \frac{1}{2 \times 2 \times 2 \times 2 \times 2}$$

$$= \underline{\underline{\frac{1}{32}}}$$

ii. $\frac{1}{5^{-2}} = 5^2$

$$= \underline{\underline{25}}$$

Example 2

Simplify: $\dfrac{2x^{-2} \times 2x^3}{2x^{-4}}$

$$\frac{2x^{-2} \times 2x^3}{2x^{-4}} = \frac{2 \times x^{-2} \times 2 \times x^3}{2 \times x^{-4}}$$

$$= \frac{\overset{1}{\cancel{2}} \times x^4 \times 2 \times x^3}{\underset{1}{\cancel{2}} \times x^2} \qquad \left(\text{since } x^{-2} = \frac{1}{x^2} \text{ and } \frac{1}{x^{-4}} = x^4\right)$$

$$= \frac{2x^7}{x^2}$$

$$= 2x^{7-2}$$

$$= \underline{\underline{2x^5}}$$

Exercise 12.3

1. Write each of the following with positive indices.

i. 3^{-4} **ii.** x^{-5} **iii.** $2x^{-1}$ **iv.** $5a^{-2}$ **v.** $5p^2q^{-2}$

vi. $\dfrac{1}{x^{-5}}$ **vii.** $\dfrac{3}{a^{-2}}$ **viii.** $\dfrac{2x}{x^{-4}}$ **ix.** $\dfrac{a}{2b^{-3}}$ **x.** $\dfrac{m}{(2n)^{-2}}$

xi. $\dfrac{t^{-2}}{m}$ **xii.** $\dfrac{p}{q^{-2}}$ **xiii.** $\dfrac{x^{-2}}{2y^{-2}}$ **xiv.** $\left(\dfrac{2x}{3y}\right)^{-2}$

2. Evaluate the following.

i. 2^{-2} **ii.** $\dfrac{1}{4^{-2}}$ **iii.** 2^{-7} **iv.** $(-4)^{-3}$ **v.** 3^{-2}

vi. $\dfrac{5}{5^{-2}}$ **vii.** 10^{-3} **viii.** $\dfrac{3^{-2}}{4^{-2}}$

3. Simplify and write the answers with positive indices.

i. $a^{-2} \times a^{-3}$ **ii.** $a^2 \times a^{-3}$ **iii.** $\dfrac{a^2}{a^{-5}} \times a^{-8}$ **iv.** $2a^{-4} \times 3a^2$ **v.** $3x^{-2} \times 4x^{-2}$

vi. $\dfrac{10x^{-5}}{5x^2}$ **vii.** $\dfrac{4x^{-3} \times x^{-5}}{2x^2}$ **viii.** $\dfrac{(2p)^{-2} \times (2p)^3}{(2p)^4}$

12.4 Zero index

A power of which the index is zero, is known as a power with zero index. 2^0 is an example of a power with zero index.

When $x^5 \div x^5$ is simplified using the laws of indices we obtain,

$$x^5 \div x^5 = x^{5-5} = x^0$$

When it is expanded and simplified we obtain, $x^5 \div x^5 = \frac{x \times x \times x \times x \times x}{x \times x \times x \times x \times x}$

$$= 1$$

Since the answers obtained when $x^5 \div x^5$ is simplified by the two methods should be the same, we obtain $x^0 = 1$.

$x^0 = 1$ where, x is any number except 0.

This result is also used when simplifying algebraic expressions.

Note:

Example 1

Simplify.

i. $\frac{x^0 \times x^7}{x^2}$

$$\frac{x^0 \times x^7}{x^2} = 1 \times x^7 \div x^2$$

$$= 1 \times x^{7-2}$$

$$= \underline{\underline{x^5}}$$

ii. $\left(\frac{x^5 \times x^2}{a}\right)^0$

$$\left(\frac{x^5 \times x^2}{a}\right)^0 = \underline{\underline{1}}$$

(Since the whole term within brackets is the base, and 0 is the index, its value is 1.)

Let us improve our skills in simplifying expressions which contain powers with zero index by doing the following exercise.

Exercise 12.4

1. Simplify the following expressions.

i. $x^8 \div x^8$ **ii.** $(2p)^4 \times (2p)^{-4}$ **iii.** $\frac{a^2 \times a^3}{a \times a^4}$

iv. $\frac{y^4 \times y^2}{y^6}$ **v.** $\frac{p^3 \times p^5 \times p}{p^6 \times p^3}$ **vi.** $\frac{x^{-2} \times x^{-4} \times x^6}{y^{-2} \times y^8 \times y^{-6}}$

2. Evaluate the following.

i. $2^0 \times 3$ **ii.** $(-4)^0$ **iii.** $\left(\frac{x}{y}\right)^0 + 1$ **iv.** $\left(\frac{x^2}{y^2}\right)^0$ **v.** $5^0 + 1$

vi. $\left(\frac{2}{3}\right)^0$ **vii.** $(2ab)^0 - 2^0$ **viii.** $(abc)^0$

12.5 Power of a power

$(x^2)^3$ is the third power of x^2. A power of this type is known as a power of a power. We can simplify this as follows.

$$\begin{aligned}(x^2)^3 &= x^2 \times x^2 \times x^2\\ (x^2)^3 &= (x \times x) \times (x \times x) \times (x \times x)\\ &= x \times x \times x \times x \times x \times x\\ &= x^6\end{aligned}$$

Therefore, $(x^2)^3 = x^6$.
Observe that the index 6 is 3 twos; that is, 2 × 3. Therefore, we can write $(x^2)^3 = x^{2 \times 3} = x^6$.
Hence, when simplifying an algebraic expression of a power of a power, the indices are multiplied.
This is also a law of indices which can be expressed as follows.

$$(a^m)^n = a^{m \times n} = a^{mn}$$

Example 1

Simplify the following.

i. $(a^5)^2 \times a$ **ii.** $(p^3)^4 \times (x^2)^0$ **iii.** $(2x^2y^3)^2$

i. $$\begin{aligned}(a^5)^2 \times a &= a^{5 \times 2} \times a\\ &= a^{10} \times a^1\\ &= a^{10+1}\\ &= a^{11}\end{aligned}$$

ii. $$\begin{aligned}(p^3)^4 \times (x^2)^0 &= p^{3 \times 4} \times x^{2 \times 0}\\ &= p^{12} \times x^0\\ &= p^{12} \times 1\\ &= p^{12}\end{aligned}$$

iii. $$\begin{aligned}(2x^2y^3)^2 &= (2 \times x^2 \times y^3)^2\\ &= 2^2 \times x^4 \times y^6\\ &= 4x^4y^6\end{aligned}$$

Let us improve our skills in simplifying expressions which contain the power of a power by doing the following exercise.

Exercise 12.5

1. Evaluate the following.

i. $(2^4)^2$ **ii.** $(3^2)^{-1}$ **iii.** $(2^3)^2 + 2^0$

iv. $(5^2)^{-1} + \frac{1}{5}$ **v.** $(4^0)^2 \times 1$ **vi.** $(10^2)^2$

2. Simplify the express using positive indices.

i. $(x^3)^4$ **ii.** $(p^{-2})^2$ **iii.** $(a^2 b^2)^2$ **iv.** $(2x^2)^3$

v. $\left(\frac{x^5}{x^2}\right)^3$ **vi.** $\left(\frac{a^3}{b^2}\right)^2$ **vii.** $\left(\frac{m^3}{n^2}\right)^{-2}$ **viii.** $(p^{-2})^{-4}$

ix. $(a^0)^2 \times a$

Miscellaneous Exercise

1. Evaluate the following.

i. $5^3 \times 5^2$ **ii.** $5^3 \div 5^2$ **iii.** $5^0 \times 5 \times 5^2$

iv. $(5^{-1})^2$ **v.** $\{(5^2)^0\}^4$ **vi.** $\frac{5^3 \times 5^{-1}}{(5^2)^2}$

vii. $5^2 \div 10^2$ **viii.** $5^2 \times 10^3 \times 5^{-1} \times 10^{-2}$

2. Simplify the following.

i. $(2x^5)^2$ **ii.** $(2ab^2)^3$ **iii.** $2x \times (3x^2)^2$ **iv.** $\frac{(4p^2)^3}{(2p^2q)^2}$

v. $\frac{(2p^2)^3}{3pq}$ **vi.** $\frac{(2a^2)^2}{5b^3} \times \frac{(3b^2)^2}{2a}$

Summary

- $a^m \times a^n = a^{m+n}$
- $a^m \div a^n = a^{m-n}$
- $x^n = \frac{1}{x^{-n}}$
- $(a^m)^n = a^{m \times n} = a^{mn}$
- $x^0 = 1$ where $x \neq 0$.

13

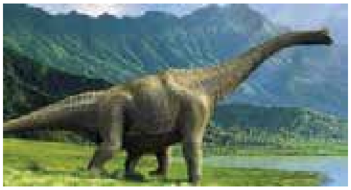
Rounding off and Scientific Notation

By studying this lesson you will be able to;

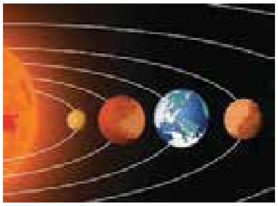
- identify the scientific notation and write numbers up to the millions period in scientific notation,
- convert numbers expressed in scientific notation to normal form,
- identify the rules related to rounding off numbers,
- round off a given number to the nearest ten, nearest hundred, nearest thousand and nearest decimal place,
- solve problems related to rounding off.

Introduction

- It is the opinion of scientists that dinosaurs are a species of animals that lived on earth 140 000 000 years ago.



- The atomic radius of the Hydrogen atom is 0.000 000 000 053 m.
- The distance from the sun to the earth is 149 600 000 000 m.



- The speed of light is 299 790 000 meters per second.

The above are four instances where numbers have been used to provide information. Using the information in the last two statements, let us find the time taken for a light

ray from the sun to approach earth.
Time =149 600 000 000 ÷ 299 790 000 seconds.

Since there are many digits in each of these numbers, they are lengthy. Therefore more space is required to write them and computations such as the above become difficult. Since a calculator can display only a limited number of characters, it is difficult to do such calculations even with an ordinary calculator. Therefore the need arises to represent such numbers in a more concise way to facilitate calculations.

In this lesson we will learn a method of writing these numbers in a concise way so that it is easy to manipulate them. Let us first do the below given review exercise to recall the facts that have been learnt in previous grades which are relevant to this lesson.

Review Exercise

1. Complete the following table.

Number	As a power of 10
1	1 = 10^0
10	10 = 10^1
100	10 × 10 = $10^{\cdots}$
1000	.... × .... ×......... = $10^{\cdots}$
10000	.... = $10^{\cdots}$
100000	.... = ...
....	.... = 10^6
....	10 × 10 × 10 × 10 × 10 × 10 × 10 = ...

2. Fill in the table given below with the following numbers according to the instructions given in the table.

5.37, 87.5, 0.75, 4.02, 1.01, 10.1, 4575, 0.07, 9, 12.3, 2.7, 9.9

Numbers that are between 1 and 10	
Numbers that are not between 1 and 10	

13.1 Scientific Notation

> The number of students sitting for the G.C.E (O/L) examination this year exceeds 700 000.
>
> \- A news item

Several ways in which the six digit number mentioned in the above news item can be expressed are given below.

i. $700 \times 1000 \longrightarrow 700 \times 10^3$

ii. $70 \times 10\,000 \longrightarrow 70 \times 10^4$

iii. $\boxed{7 \times 100\,000 \longrightarrow 7 \times 10^5}$

From the above, the last form is used often as it can be easily written and is the most concise form. It is a product of two parts. The first part is 1 or a number between 1 and 10 while the second part is a power of 10.

$$\underset{\substack{\uparrow \\ \text{Number between} \\ \text{1 and 10 or 1}}}{7} \times \underset{\substack{\nwarrow \\ \text{Power of 10}}}{10^5}$$

Writing a number with many digits in this manner as a product of two numbers, where one is between 1 and 10 or 1 and the other is a power of 10, is known as the scientific notation.

> If A is a number between 1 and 10 or 1 and n is an integer, then $A \times 10^n$ is a number written in scientific notation (Here $1 \leq A < 10$).

Let us write 280 000 in scientific notation.

Taking the first couple of digits in 280 000 and writing it as a number between 1 and 10 we get 2.8.

$$\begin{aligned} \therefore 280\,000 &= 2\,\widetilde{80000} \\ &= 2.8 \times 100\,000 \\ &= 2.8 \times 10^5 \end{aligned}$$

Therefore 280 000 expressed in scientific notation is 2.8×10^5.

Example 1

Write the following numbers in scientific notation.

a. 20 000 **b.** 4240 **c.** 1 million **d.** 3.47 **e.** 34.7
f. 6 **g.** 289.325 **h.** 2491.32

a. $20\,000 = 2.0 \times 10\,000$
$= \underline{\underline{2 \times 10^4}}$

b. $4240 = 4.24 \times 1000$
$= \underline{\underline{4.24 \times 10^3}}$

c. 1 million $= 1000\,000$
$= \underline{\underline{1 \times 10^6}}$

d. $3.47 = 3.47 \times 1$
$= \underline{\underline{3.47 \times 10^0}}$ (since$1 = 10^0$)

e. $34.7 = 3.47 \times 10$
$= \underline{\underline{3.47 \times 10^1}}$

f. $6 = 6 \times 1$
$= \underline{\underline{6 \times 10^0}}$

g. $289.325 = 2.89325 \times 100$
$= \underline{\underline{2.89325 \times 10^2}}$

h. 2491.32

$2491.32 = 2.49132 \times 10^3$

By shifting the decimal point 3 places to the left, we obtain 2.49132×10^3.

Exercise 13.1

1. Complete the following table according to the given examples.

	Number	1 or a number between 1 and 10 × a power of 10	Scientific notation
	48	4.8×10	4.8×10^1
a.	8		
b.	99		
c.	78		
	548	5.48×100	5.48×10^2
d.	999		
e.	401		
f.	111		
	34 700	3.47×10000	3.47×10^4
g.	54 200		
h.	49 40000		
i.	10 00000		

2. Write each of the following numbers in scientific notation.

a.	200	**f.**	340000
b.	254	**g.**	6581200
c.	1010	**h.**	7.34
d.	5290	**i.**	18.5
e.	74300	**j.**	715.8

3. A few important facts about Sri Lanka are given below. Write the numbers which are related to these facts in scientific notation.

The height of Piduruthalagala mountain is 2524 m.
The area of Sinharaja forest is 9300 hectares.
The length of Mahaweli river is 335 km.
The total area of Sri Lanka is 65 610 km^2.

13.2 Writing a number between 0 and 1 in scientific notation

Consider the pattern given below.

$$10\,000 = 10^4$$
$$1000 = 10^3$$
$$100 = 10^2$$
$$10 = 10^1$$
$$1 = 10^0$$
$$0.1 = \frac{1}{10} = \frac{1}{10^1} = 10^{-1}$$
$$0.01 = \frac{1}{100} = \frac{1}{10^2} = 10^{-2}$$
$$0.001 = \frac{1}{1000} = \frac{1}{10^3} = 10^{-3}$$

It is clear that,

when writing 0.1 as a power of 10 the index is –1

when writing 0.01 as a power of 10 the index is –2

when writing 0.001 as a power of 10 the index is –3.

0.75 is a number which is less than 1. When it is written in terms of a number between 1 and 10, it should be written as 7.5 divide by 10. The way this is done mathematically can be expressed as follows.

Since $0.75 \times 10 = 7.5$,

$$0.75 = \frac{7.5}{10}$$

$$= \frac{7.5}{10^1} \quad \text{(Since } 10 = 10^1\text{)}$$

$$= \underline{\underline{7.5 \times 10^{-1}}} \quad \left(\text{Since } \frac{1}{10^1} = 10^{-1}\right)$$

Accordingly, the number 0.75 has been expressed as the product of a number between 1 and 10 and a power of 10.

$\therefore$ 0.75 expressed in scientific notation is 7.5×10^{-1} .

In the same manner, let us write 0.0034 in scientific notation.

Since $0.0034 \times 1000 = 3.4$,

$$0.0034 = \frac{3.4}{1000}$$

$$= \frac{3.4}{10^3}$$

$$= \underline{\underline{3.4 \times 10^{-3}}}$$

Note: When a number between 0 and 1 is written in scientific notation, the index of the power of 10 is a negative integer.

Example 1

Express each of the following numbers in scientific notation.

a. 0.8453 **b.** 0.047 **c.** 0.000017

a. $0.8453 = 8.453 \div 10$

$$= \frac{8.453}{10}$$

$$= \frac{8.453}{10^1}$$

$$= \underline{\underline{8.453 \times 10^{-1}}}$$

b. $0.047 = 4.7 \div 100$

$$= \frac{4.7}{100}$$

$$= \frac{4.7}{10^2}$$

$$= \underline{\underline{4.7 \times 10^{-2}}}$$

c. 0.000017

$$= 1.7 \div 100000$$

$$= \frac{1.7}{10^5}$$

$$= \underline{\underline{1.7 \times 10^{-5}}}$$

Exercise 13.2

1. Copy the following table and complete it.

Number less than 1	Expressed in terms of a number between 1 and 10	Scientific notation
a. 0.041	$\frac{4.1}{100} = \frac{4.1}{10^2}$	4.1×10^{-2}
b. 0.059		
c. 0.0049		
d. 0.000 135	$\frac{1.35}{10000} = \frac{1.35}{10^4}$	$\ldots\ldots\ldots \times 10^{-4}$
e. 0.000 005		
f. 0.000 003 9		
g. 0.111345		

2. Write each of the following numbers in scientific notation.

a. 0.08
b. 0.543
c. 0.0004
d. 0.0019
e. 0.00095
f. 0.000 000 054

3. Express each of the following numbers in scientific notation.

The radius of an atom is 0.000 0000 01 cm.
The mass of one cubic centimetre of air is 0.00129 g.
The mass of one cubic centimetre of hydrogen is 0.000 088 9 g.

13.3 Converting numbers expressed in scientific notation to general form

As an example, let us convert the number 5.43×10^4 written in scientific notation to general form.

Method I

$5.43 \times 10^4 = 5.43 \times 10000$
$= 54\,300$
$\therefore 5.43 \times 10^4 = 54300$

Method II

Since it is multiplied by 10^4, (that is 10 000) shifting the decimal point 4 places to the right, we obtain 54300.

54 300
54 300

Another example is given below. This is an instance where the index of the power of 10 is a negative number.

Method I

$$5.43 \times 10^{-4} = 5.43 \times \frac{1}{10^4}$$
$$= 5.43 \div 10000$$
$$= 0.000543$$

Method II

Since it is divided by 10^4, shifting the decimal point 4 places to the left we obtain 0.000543.

0.000543

Example 1

Convert the following numbers to general form.

(i) 8.9×10^3

(i) $8.9 \times 10^3 = 8.9 \times 1000$

$= \underline{\underline{8900}}$ 8900.

(ii) 8.9×10^{-3}

(ii) $8.9 \times 10^{-3} = 8.9 \times \frac{1}{10^3}$

$= \underline{\underline{0.0089}}$ 0.0089

Here for example, 8.9×10^3 can be directly written as 8 900. When multiplying, if the index of the power of 10 is a positive integer, then the decimal point should be shifted to the right, the same number of positions as the index (adding zeros if necessary). When multiplying, if the index of the power of 10 is a nagative integer, then the decimal point should be shifted to the left, the same number of positions as the index.

Exercise 13.3

1. Fill in the given blanks to convert each of the following numbers expressed in scientific notation to general form.

i. $5.43 \times 10^3 = 5.43 \times$ ....
$= \underline{\underline{\text{...........}}}$

ii. $7.25 \times 10^5 =$ .... $\times$ ....
$=$ ....

iii. $6.02 \times 10^1 =$ .... $\times$ ....
$=$ ....
$\underline{\underline{\qquad\qquad}}$

iv. $5.99 \times 10^{-2} = 5.99 \times \frac{1}{10^{\cdots}}$
$= \frac{5.99}{\text{.......}}$
$= \underline{\underline{0.0599}}$

v. $1.06 \times 10^{-6} = 1.06 \times$ ....
$= \frac{1.06}{\text{.......}}$
$= \underline{\underline{\text{...........}}}$

2. Convert the following numbers to general form.

a. 8.9×10^2
b. 1.05×10^4
c. 7.994×10^5
d. 8.02×10^3
e. 9.99×10^7
f. 7.2×10^{-1}
g. 8.34×10^{-3}
h. 5.97×10^{-4}
i. 9.12×10^{-5}
j. 5.00×10^{-6}

3. Select the larger number from each of the number pairs given below.

a. 2.1×10^4, 3.7×10^4
b. 2.1×10^4, 3.7×10^3
c. 2.1×10^4, 3.7×10^5
d. 2.1×10^4, 2.1×10^{-4}
e. 2.1×10^4, 3.7×10^{-3}
f. 2.1×10^{-4}, 3.7×10^{-3}

4. Write the following numbers in general form.

The area of the earth covered by land is 1.488×10^8 km^2.
The area of the earth covered by the oceans is 3.613×10^8 km^2.
The total surface area of the earth is 5.101×10^8 km^2.

Rounding off numbers

> It is reported that 2 500 people attended the book exhibition held at Sarasvathi hall over the weekend.
>
> \- A news item

The number of tickets that were sold over the weekend to those who attended the exhibition mentioned in the news item was 2 483. Accordingly, the actual number of people who attended the exhibition is 2 483. The number 2 500 which is mentioned in the news item is a number which is close to 2 483, easy to remember and has a special feature. Moreover it is sufficient to communicate an idea of the number that attended the exhibition.

Rounding off a number means representing the value of the number by a value which is close to it, which is simple and, easy to remember and communicate. There are many ways of rounding off numbers. Let us consider a few of them.

13.4 Rounding off to the nearest 10

Representing a number by the multiple of 10 which is nearest to it is known as "rounding off to the nearest 10".

Let us round off 2 483, which is the number of people who attended the exhibition, to the nearest 10. The number 2 483 lies between the two multiples of ten, 2 480 and 2 490. However it is closer to 2 480 than to 2 490. Accordingly, when 2 483 is rounded off to the nearest 10, we obtain 2 480.

We can describe this more generally as follows.

When rounding off 2 481, 2 482, 2 483 and 2 484 to the nearest 10 we obtain 2 480. This is because the multiple of 10 which all these numbers are closest to is 2 480. Similarly, if we round off 2 486, 2 487, 2 488 and 2 489 to the nearest 10 we obtain 2 490. The reason for this is also the same as the above. Even though the remaining number 2 485 is at an equal distance from the two multiples of ten 2 480 and 2 490, when rounding it off to the nearest 10, the convention is to round it off to the nearest 10 which is greater than it, that is, to 2 490. Finally, it is clear that when 2 480 is rounded off to the nearest 10 we obtain 2 480 itself and when 2 490 is rounded off to the nearest 10 we obtain 2 490 itself.

Example 1

Round off to the nearest 10.

i. 273 **ii.** 1428 **iii.** 7196.

i. 270

ii. 1430

iii. 7200

Exercise 13.4

1. Round off each of the following numbers to the nearest 10.

a. 33
b. 247
c. 3 008
d. 59
e. 306
f. 4 010
g. 85
h. 1514
i. 1 895
j. 12 345
k. 234 532
f. 997 287

2. The height of the mountain Piduruthalagala is 2 524 m. Round off this number to the nearest 10.

3. Write every whole number which when rounded off to the nearest 10 is equal to 140.

4. Write every whole number which when rounded off to the nearest 10 is equal to 80.

What is the smallest whole number which when rounded off to the nearest 10 is 80?
What is the largest whole number which when rounded off to the nearest 10 is 80?

5. When a certain number is rounded off to the nearest 10, the number 260 is obtained. Find separately the least and the greatest value that this number can take.

13.5 Rounding off to the nearest 100 or 1000

"Rounding off to the nearest 100" or "to the nearest 1000" is defined in the same way that "rounding off to the nearest 10" was defined.

For example, the number 7 346 is between the two multiples of hundred, 7 300 and 7 400 and is closer to 7 300 than to 7 400. Therefore when 7 346 is rounded off to the nearest 100, we obtain 7 300. Similarly, if we round off 7 675 to the nearest 100 we obtain 7 700. In general, if we round off a number from 7 300 to 7 349 (both included) to the nearest 100 we obtain 7 300, and if we round off a number from 7 350 to 7 400 (both included) to the nearest 100 we obtain 7 400.

Now, let us consider how to round off numbers to the nearest 1000. For example, when 41 873 is rounded off to the nearest 1000 we obtain 42 000. The reason for this is because 41 873 is closer to 42 000 than to 41 000.

It must be clear to you by this time, what occurs when we round off numbers. Now let us consider a method that can be used to round off numbers easily.

- Let us round off 2 425 to the nearest 100.

2425

The two multiples of 100 between which 2425 lies are 2400 and 2500. The value of 2425 is less than the value of 2450 which is exactly at the centre between these two multiples of 100. Therefore, 2425 is closer to 2400 than to 2500.

Accordingly, when 2425 is rounded off to the nearest 100 we obtain 2400.

- Let us round off 2485 to the nearest 100.

2485

The two multiples of 100 between which 2485 lies are 2400 and 2500. The value of 2485 is greater than the value of 2450 which is exactly at the centre between these two multiples of 100. Therefore, 2485 is closer to 2500 than to 2400.

Accordingly, when 2485 is rounded off to the nearest 100 we obtain 2500.

- Let us round off 2450 to the nearest 100.

2450

The two multiples of 100 between which 2450 lies are 2400 and 2500. The number 2450 is exactly at the centre between these two multiples of 100. According to the convention, the number which is at the centre is rounded off to the nearest multiple of 100 greater than that number.

Accordingly, when 2450 is rounded off to the nearest 100 we obtain 2500.

- Let us round off 2485 to the nearest 1000.

2485

The two multiples of 1000 between which 2485 lies are 2000 and 3000. The value of 2485 is less than the value of 2500 which is exactly at the centre between these two multiples of 1000. Therefore, 2485 is closer to 2000 than to 3000.

Accordingly, when 2485 is rounded off to the nearest 1000 we obtain 2000.

- Let us round off 2754 to the nearest 1000.

2754

The two multiples of 1000 between which 2754 lies are 2000 and 3000. The value of 2754 is greater than the value of 2500 which is exactly at the centre between these two multiples of 1000. Therefore, 2754 is closer to 3000 than to 2000.

Accordingly, when 2754 is rounded off to the nearest 1000 we obtain 3000.

- Let us round off 12 500 to the nearest 1000.

12500

The two multiples of 1000 between which 12 500 lies are 12 000 and 13 000. The number 12 500 is exactly at the centre between these two multiples of 1000. According to the convention, the number which is at the centre is rounded off to the nearest multiple of 1000 greater than that number.

Accordingly, when 12 500 is rounded off to the nearest 1000 we obtain 13 000.

Exercise 13.5

1. Round off each of the following numbers to the nearest 100.

a. 54 **b.** 195 **c.** 1009 **d.** 2985 **e.** 72324 **f.** 7550

2. Round off each of the following numbers to the nearest 1000.

a. 1927 **b.** 2433 **c.** 19999 **d.** 45874 **e.** 38000 **f.** 90500

3. The number of students in a school is 2 059. Round off this number to the,
i. nearest 10
ii. nearest 100
iii. nearest 1000.

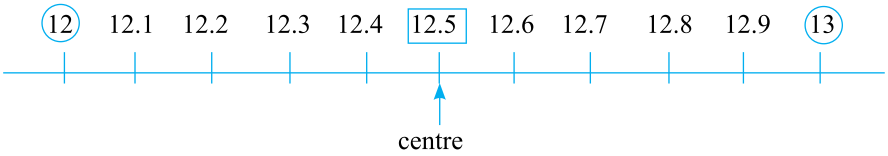
4. When a number is rounded off to the nearest 100, the number 4 500 is obtained.
i. What is the smallest whole number it could be?
ii. What is the largest whole number it could be?

Rounding off decimal numbers

When the mass of a 5 year old child was measured, it was 12.824 kg. If we write this in grammes, it is 12 824 g. This value was obtained because the scale used for this purpose gives the mass to the nearest gramme. However, for practical purposes the mass is usually required to the nearest kilogramme, to the nearest 10th of a kilogramme or to the nearest 100th of a kilogramme.

It is useful to know how to round off any given decimal number to the nearest whole number, nearest first decimal place, nearest second decimal place, etc. In this lesson we will learn how to round off decimal numbers.

Initially, let us consider how to round off a number with one decimal digit to a whole number.
Let us round off 12.7 to the nearest whole number.



The whole numbers on either side of 12.7 are 12 and 13.

Since the numbers 12.1, 12.2, 12.3 and 12.4 are closer to the whole number 12 than to the whole number 13, when these numbers are rounded off to the nearest whole number we obtain 12. Similarly, since the numbers 12.6, 12.7, 12.8 and 12.9 are closer to 13 than to 12, when these numbers are rounded off to the nearest whole number we obtain 13. Furthermore, as in the above sections, 12.5 rounded off to the nearest whole number is accepted by convention to be 13. Accordingly, when 12.7 is rounded off to the nearest whole number we obtain 13.

Similarly,
12.3 rounded off to the nearest whole number is 12 and
12.5 rounded off to the nearest whole number is 13.

Rounding off to a given decimal place

Round off 3.74 to the nearest first decimal place.

In 3.74, the digit in the first decimal place is 7 and the digit in the second decimal place is 4. When rounding off to the first decimal place, the digit in the second decimal place is considered and the digit in the first decimal place is adjusted accordingly if necessary.
The rule used here for rounding off is similar to that used in the previous sections. Since the number with one decimal digit which is closest to the numbers 3.71, 3.72, 3.73 and 3.74 is 3.7, when these numbers are rounded off to the first decimal place we obtain 3.7. Similarly when the numbers 3.75, 3.76, 3.77, 3.78 and 3.79 are rounded off to the first decimal place we obtain 3.8. Accordingly, 3.74 rounded off to the first decimal place is 3.7.

The rule for rounding off numbers to other decimal places is also the same. Let us consider the following example.

Example 2

Round off

i. 3.784 **ii.** 3.796

to the nearest second decimal place.

When rounding off to the nearest second decimal place, the digit in the third decimal place needs to be considered.

i. 3.784 lies between 3.78 and 3.79. Since 3.784 is closer to 3.78 than to 3.79, when it is rounded off to the nearest second decimal place we obtain 3.78.

ii. 3.796 lies between 3.79 and 3.80. Since 3.796 is closer to 3.80 than to 3.79, when it is rounded off to the nearest second decimal place we obtain 3.80.

Exercise 13.6

1. Round off each of the following numbers to the nearest whole number and to the nearest first decimal place.

i. 5.86 **ii.** 12.75 **iii.** 10.43 **iv.** 123.79

v. 8.04 **vi.** 13.99 **vii.** 101.98 **viii.** 100.51

2. The value of π is 3.14159… . Round off this value to

i. the nearest whole number
ii. the nearest first decimal place
iii. the nearest second decimal place.

3. The diameter of a sphere is 3.741 cm. Round off this value to

i. the nearest first decimal place
ii. the nearest second decimal place.

4. According to a survey plan, the area of a plot of land is 0.785 ha. Round off this value to

i. the nearest first decimal place
ii. the nearest second decimal place.

5. In an animal farm, the mean amount of milk obtained from a healthy cow per day is 5.25 *l*. If there are 45 such animals, round off the amount of milk obtained per a day

i. to the nearest litre

ii. to the nearest first decimal place.

Miscellaneous Exercise

1. Write each of the following groups of numbers in ascending order.

i. 3.10×10^{2} , 3.10×10^{-4} , 3.10×10^{0} , 3.10×10^{5}

ii. 4.78×10^{-2} , 1.43×10^{4} , 9.99×10^{-3} , 2.32×10^{1}

iii. 7.85×10^{0} , 7.85×10^{-4} , 7.85×10^{2} , 7.85×10^{-2}

2. There are 250 labourers working in a factory which pays Rs 1 230 per day as wages to a labourer.

i. Find the amount of money required per day to pay the wages of all these labourers.

ii. Write 1 230 and 250 in scientific notation.

iii. Using the numbers written in (ii) above in scientific notation, find the amount of money required per day for wages.

iv. Compare the values obtained in (i)and (iii) above.

3. The volume of tea produced in a day at a certain tea factory is 1 500 kg. If the factory operates for 30 days during a certain month, show that the volume of tea produced that month is 4.5×10^{4} kg.

4. Fill in the tables given below.

(a)

Expression	The expression obtained when the numbers in the given expression are rounded off to the nearest whole number	The value obtained for the expression by taking the product after rounding off the numbers
59.2 × 9.97	60 × 10	600
8.4 × 5.7	8 × 6	48
12.3 × 11.95	.... ×........	....
10.15 × 127.6	.... ×........	....
459.7 × 3.51	.... ×........	....
109.5 × 4.49	.... ×........	....

(b)

Expression	Product without rounding off the numbers	Value obtained by rounding off the product to the nearest whole number
59.2 × 9.97	590.224	590
8.4 × 5.7		
12.3 × 11.95		
10.15 × 127.6		
459.7 × 3.51		
109.5 × 4.49		

Summary

- Scientific notation is a method of expressing a number concisely to facilitate calculations.
- If $1 \leq A < 10$ and $n \in \mathbb{Z}$ then $A \times 10^n$ is a number expressed in scientific notation.

14

Loci and Constructions

By studying this lesson you will be able to,

- identify four basic loci,
- construct a line perpendicular to a given line,
- construct the perpendicular bisector of a straight line segment,
- construct and copy angles,
- solve problems related to loci and constructions.

Loci

A few motions that you can observe in the environment are given below. Let us consider the path of each motion.

1. Cotton floating in the air
2. A bird flying
3. A ball hit by a bat
4. A fruit falling from a tree
5. The tip of a hand of a working watch
6. A child riding a see-saw

You may observe that even though the motions of 1 and 2 are complex and unpredictable, the motions of 3 to 6 have a definite path. It is important to learn about loci in geometry to develop a proper understanding of the paths of the objects undergoing these motions.

A set of points satisfying one or more conditions is known as a locus.

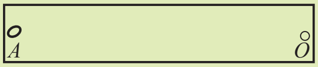
14.1 Basic Loci

Now let us consider four basic loci.

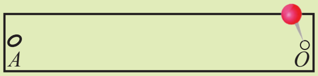
1. The locus of points which are at a constant distance from a fixed point.

Activity 1

Step 1: Take a strip of cardboard of length 5 cm, make two small holes near the two ends and name them *O* and *A*.

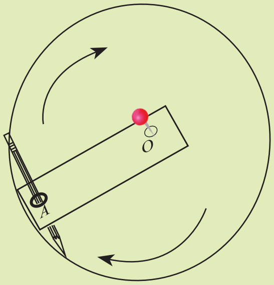


Step 2: Keep the above strip of cardboard on a piece of paper, place a pin through the hole *O* and keep it firm on the piece of paper.



Step 3: Place a pencil point through the hole *A* and while holding the pin tightly so that it doesn't move, move the pencil and mark the path it takes.

Step 4: At the end of the activity, identify the locus that is obtained.



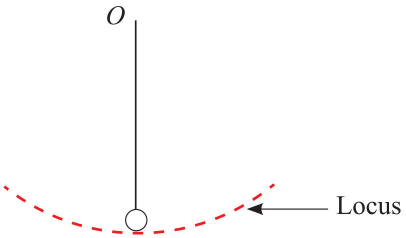
In the above activity, you would have obtained a circular path. Accordingly,

The locus of points on a plane which are at a constant distance from a fixed point is a circle.

Example 1

Draw a sketch of the locus of the bottommost point of the bob of a pendulum in a working pendulum clock.

The locus relevant to this motion is a part of a circle with centre the fixed point of the rod/string to which the bob is connected and radius the distance from this fixed point to the bottommost point of the bob.

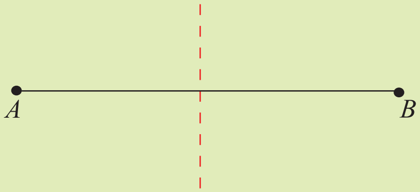


2. The locus of points which are equidistant from two fixed points.

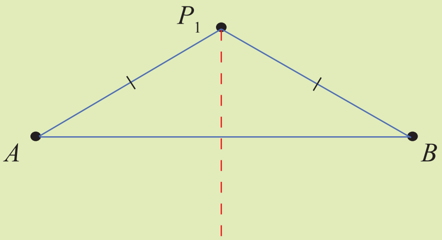
Activity 2

Step 1: Draw a straight line segment of length 10 cm on an oil paper/tissue paper and name it AB.

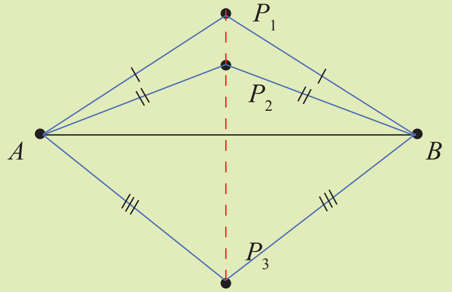
Step 2: Identify the axis of symmetry of the straight line AB by folding the tissue paper such that the two points A and B coincide and mark it with a dashed line.



Step 3: Mark a point P_1 on the dashed line, draw the straight lines P_1A and P_1B, and measure and write their lengths.



Step 4: Mark several more points on the dashed line and measure and write the distances from the points A and B to each of these points.



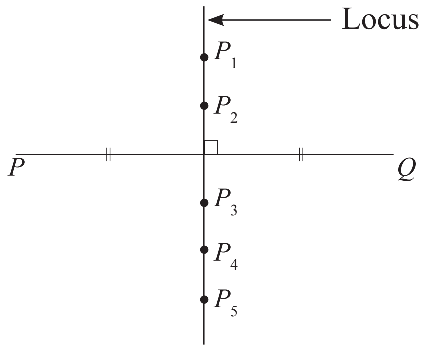
Step 5: Check whether the distances from the points A and B to each of these points on the dashed line are equal and write your conclusion.

When folding the paper as above, such that A and B coincide, observe that the fold line obtained is perpendicular to AB and that it passes through the midpoint of AB. This line is called the perpendicular bisector of the line segment AB. Observe further that the distances from the points A and B to any point you chose on the perpendicular bisector are equal.

The locus of points which are equidistant from two given points is the perpendicular bisector of the line joining the two points.

Example 2

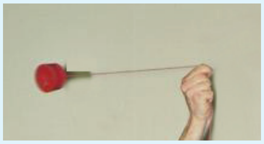
Draw a rough sketch of the locus of points which are equidistant from the two given points P and Q. Name five points on the locus as P_1, P_2, P_3, P_4 and P_5.



Exercise 14.1

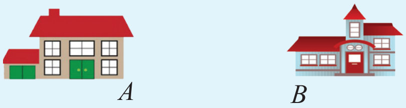
1. By a rough sketch, show the locus relevant to each motion given below.

a. The path of a rubber bushing which is tied to one end of a rope of length 50 cm and rotated, by holding the stretched rope at the other end as shown in the figure



b. The path of the tip of a hand of a working clock

c. The figure shows two houses located 50 m from each other on horizontal ground. It is required to build a wall exactly halfway between the two houses (Points A and B). Indicate by a rough sketch where the wall should be built.



c. The path of the fire of a torch held by a fire torch rotating dancer in a perahera (while the fire torch rotator is stationery)

d. The path of a person riding a Ferris wheel

e. The paths of the two children riding a see-saw while sitting at the two ends of the see-saw

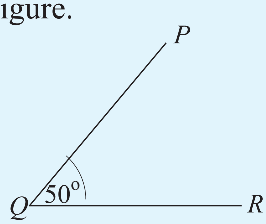
2. In the given figure, P and Q are two trees planted on flat ground, at a horizontal distance of 25 m from each other.

   i. It is required to fix a tap at a distance of 15 m from each tree. With the knowledge on loci, draw a rough sketch to indicate how the points where the tap can be fixed are found.

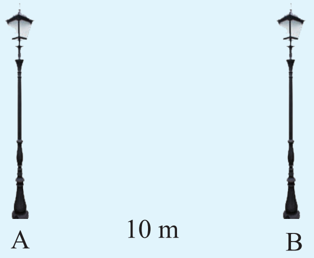
   ii. It is required to cut a drain equidistant from the trees. Draw a rough sketch to indicate the location of the drain.

3. Draw an angle of 50° and name it $P\hat{Q}R$ as shown in the figure.

   With the knowledge on loci, draw a rough sketch to indicate how the point which is equidistant from the points Q and R and lying on the arm PQ is found, and name it S.



4. A and B are two lamp posts located at a distance of 10 m from each other.



   i. It is required to fix another lamp post C at a distance of 6 m from A and 8 m from B. Mark the location of the lamp post C on a suitable rough sketch.

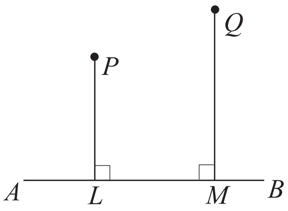
   ii. It is required to fix a lamp post D equidistant form the posts A and B. Mark the location of the lamp post D on a suitable rough sketch.

14.2 More on basic loci

3. The locus of points which are at a constant distance from a fixed line

The distance from a point to a line is the length of the perpendicular line drawn from the point to the line.
Accordingly,

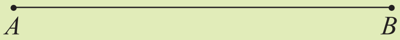
the distance from P to the line AB is the length of PL and the distance from Q to the line AB is the length of QM.



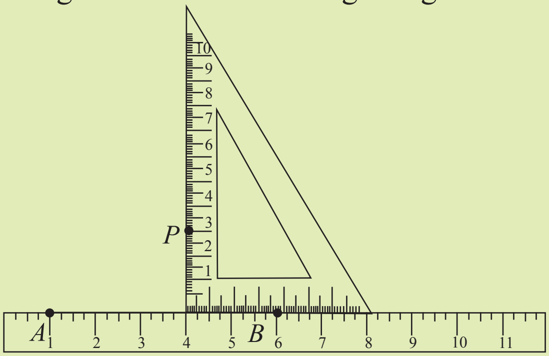
Now let us do the following activity to determine the locus of points which are at a constant distance from a fixed line.

Activity 1

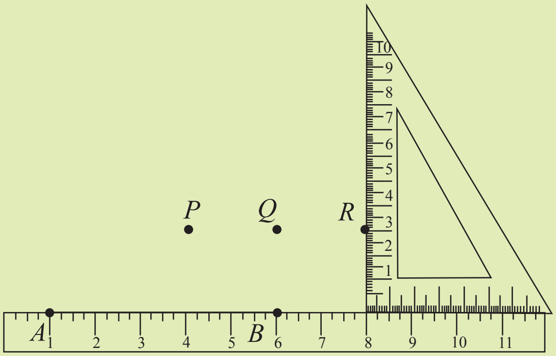
Step 1: Draw a straight line segment in your exercise book and name it *AB*.



Step 2: Place a straight edge on the line *AB* and a set square touching the straight edge as shown in the figure. Mark a point 3 cm from *AB* using the scale on the straight edge and name it *P*.



Step 3: By changing the position of the set square, mark a couple more points at a distance of 3 cm from *AB* and name them *Q* and *R*.



Step 4: Using a straight edge, join the above marked points *P*, *Q* and *R*.

Step 5: Describe the locus of points which are at a distance of 3 cm from the line *AB*. Observe that a similar locus can be drawn on the other side of *AB* too.

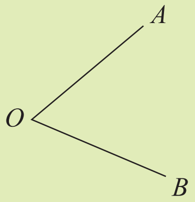
It is clear from the above activity that, the locus of a point at a constant distance of 3 cm from the line *AB* is a straight line parallel to the line *AB* and at a distance of 3 cm from it. Moreover, two loci can be drawn on either side of *AB*.

The locus of points which are at a constant distance from a straight line are the two straight lines parallel to it and at the given constant distance from it, on either side of it.

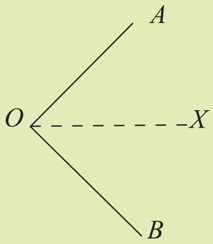
4. The locus of points equidistant from two intersecting straight lines.

Activity 2

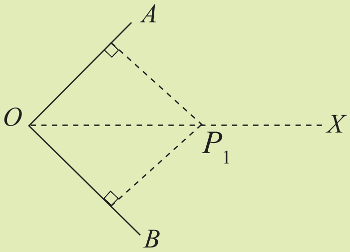
Step 1: On a transparent paper (like oil paper) draw a pair of straight lines as shown in the figure and name them *OA* and *OB*.



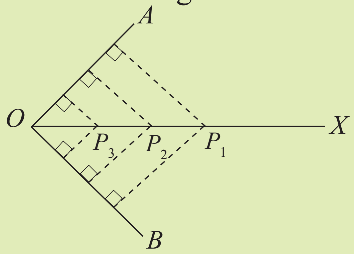
Step 2: Fold the transparent paper such that *OA* and *OB* coincide, and mark the fold line with a dotted line. Name it *OX*.



Step 3: Mark a point on the dotted line and name it P_1. Using a set square, draw two lines from P_1 perpendicular to *OA* and *OB* respectively, measure their lengths and write them down.



Step 4: Mark more points on line OX as shown in the figure and name them P_2, P_3, etc. From each of these points, draw perpendicular lines to OA and OB, measure their lengths and write them down.



Step 5: Measure $A\hat{O}X$ and $B\hat{O}X$ and write what can be concluded about the line OX.

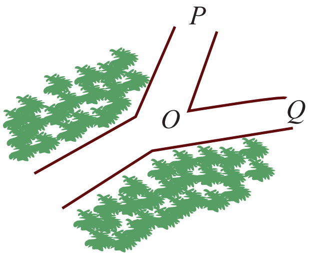
From the above activity it is clear that OX is the line that divides the angle $A\hat{O}B$ into two equal angles and that the distances from any point on the line OX to the lines OA and OB are equal.
Furthermore, since the paper was folded such that OA and OB coincide, the angles $A\hat{O}X$ and $B\hat{O}X$ are equal to each other.

OX is known as the angle bisector of $A\hat{O}B$.

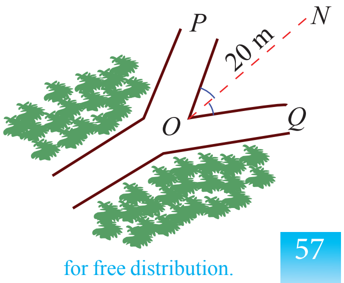
The locus of points equidistant from two intersecting straight lines is the angle bisector of the angles formed by the intersection of the two lines.

Example 1

OP and OQ are two roads which diverge at the junction O. It is required to fix a notice board at a point which is 20 m from the junction O and at an equal distance from both these roads. Using the knowledge on loci, indicate by a rough sketch how you would find the place where the notice board should be fixed.

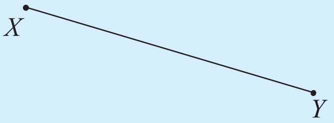


The required position (say N) should be a point on the angle bisector of $Q\hat{O}P$. Since $ON = 20$ m, N should be the point on the angle bisector a distance of 20 m from O.



Exercise 14.2

1. Draw a straight line segment and name it XY. Illustrate by a rough sketch, the locus of points which are 4 cm away from it.



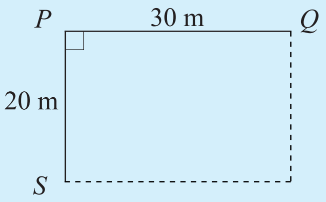
2. A student walks on a straight road rotating a wheel of diameter 20 cm which is fixed to a handle. Illustrate by a rough sketch the locus of the centre of the wheel.

3. The figure shows the positions of the hour hand and the minute hand of a clock at a certain instant. At this moment, the second hand is located at an equal distance from these two hands. Indicate the position of the second hand by a separate rough sketchs.

4. *A* drain PQ of length 50 m which is located in a certain plot of land is shown in the figure. A tap needs to be fixed at a distance of 10 m from PQ and at an equal distance from both the ends P and Q. Illustrate by a rough sketch the position/ positions where the water tap can be fixed.

5. A piece of cake cut from a round (circular) cake is shown in the figure. It is required to divide this piece of cake into two equal pieces. Using the knowledge on loci, indicate by a sketch how this piece should be cut.

6. PQ and PS are two boundaries of a rectangular plot of land. A tree needs to be planted in this plot of land such that it is 8 m from the boundary PQ and 5 m from the boundary PS. Illustrate by a rough sketch where the tree should be planted and name it T.



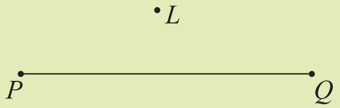
14.3 Constructing lines perpendicular to a given straight line

Let us explain two phrases that are commonly used in constructions. When drawing a circle using a pair of compasses, phrases such as "taking a certain point as the centre" and "taking a certain length as the radius" are often used. For example, "Taking point *A* as the centre" means the circle or arc should be drawn with the point of the pair of compasses kept at the point *A*; and "Taking *AB* as the radius" means that the distance between the point of the pair of compasses and the pencil point should be equal to the length of *AB*.

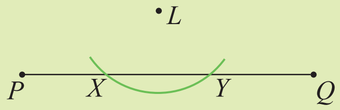
1. Constructing a line perpendicular to a given line from an external point

Activity 1

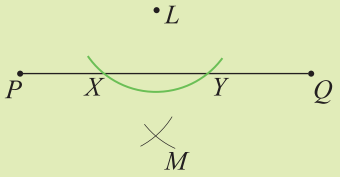
Step 1: Draw a straight line segment in your exercise book and name it *PQ*. Mark a point external to *PQ* and name it *L*.



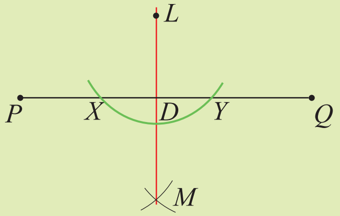
Step 2: Taking a length which is more than the distance from *L* to *PQ* as the radius and *L* as the centre, drawn arc such that it intersects the line *PQ*. Name the points of intersection *X* and *Y*.



Step 3: Taking each of the points *X* and *Y* as the centre and using the same radius, draw two arcs such that they intersect each other as shown in the figure. Name the point of intersection *M*.



Step 4: Join the points L and M and name the point at which LM intersects PQ as D. Measure and write the magnitude of $L\hat{D}P$.



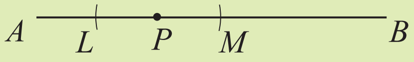
At the end of the above construction, you would have obtained that $L\hat{D}P = 90^{o}$. That is, LD is the perpendicular line drawn from the point L to the line PQ.

2. Constructing a line perpendicular to a given line through a point on the line

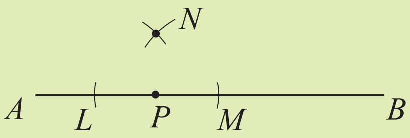
Activity 2

Step 1: Draw a straight line and name it AB. Mark a point on it and name it P.

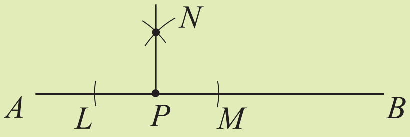
Step 2: Taking a length less than the length of PA as the radius, and taking P as the centre, draw two arcs using the pair of compasses such that they intersect the line segments PA and PB. Name the two points of intersection L and M.



Step 3: Taking a length greater than the one taken in step 2 as the radius, and taking L and M as the centres, draw two arcs such that they intersect each other as shown in the figure. Name the point of intersection N.



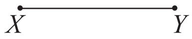
Step 4: Join *NP*, measure the magnitude of the angle $N\hat{P}A$ and write its value.



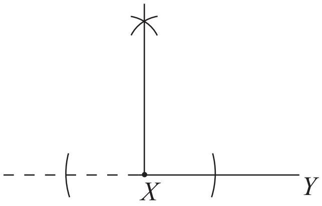
At the end of the above construction you would have obtained that $N\hat{P}A = 90^\circ$. That is, the line drawn perpendicular to *AB* through the point *P* is *PN*.

3. Constructing a line perpendicular to a given straight line segment through an end point

Let us assume that we need to draw a line perpendicular to the line segment *XY* through the point *X*.



Produce the line *YX* and do this construction using the method identified above.

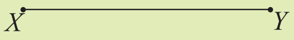


4. Constructing the perpendicular bisector of a straight line segment

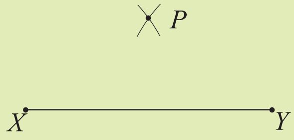
The straight line which is perpendicular to a given line segment and which passes through the midpoint of that line segment, was identified earlier as the perpendicular bisector of that line segment.

Draw a straight line segment and name it *XY*. Let us do the activity given below to construct the perpendicular bisector of this line segment

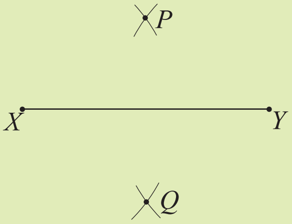
Activity 3



Step 1: Taking a length greater than half the length of XY as the radius, and without changing it, draw two arcs with X and Y as the centres, such that they intersect each other. Name the point of intersection P.

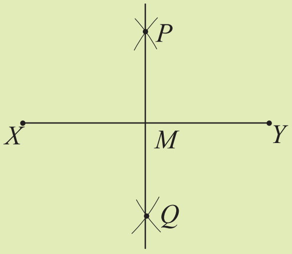


Step 2: As done above, taking X and Y as the centres, draw two other arcs such that they intersect each other on the side of XY opposite to the side on which P is located. Name the point of intersection Q.



Note: It is not necessary to use the same radius in the above two steps.

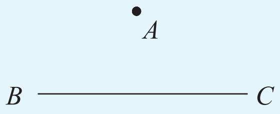
Step 3: Join PQ and name the point at which PQ intersects XY as M. Measure XM and MY and the magnitude of $X\hat{M}P$. What can be concluded regarding the line PQ?



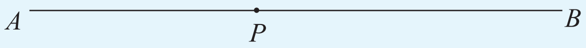
You would have identified in the above activity that $XM = MY$ and $X\hat{M}P = 90^{\circ}$. Accordingly, PQ bisects the line segment XY perpendicularly. Therefore, PQ is the perpendicular bisector of XY.

Exercise 14.3

1. Draw a straight line as shown in the figure and name it BC. Construct a perpendicular line from the point A to the line BC.



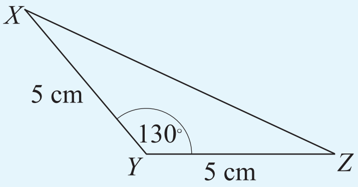
2. Draw the line AB such that AB = 7 cm. Mark the point P on AB such that AP = 3 cm and construct a line perpendicular to AB through P.



3. Draw any acute angled triangle and name it PQR.

- **i.** Construct a line perpendicular to QR from P.
- **ii.** Construct a line perpendicular to PR from Q.
- **iii.** Construct a line perpendicular to PQ from R.

4. **i.** Using a protractor, draw an angle of 130° and as shown in the figure mark 5 cm on each arm and complete the triangle XYZ.



- **ii.** Construct a perpendicular line from Y to the line XZ and name the point at which it meets XZ as D.
- **iii.** Measure and write the lengths of XD and ZD.

5. Construct a rectangle of length 6 cm and breadth 4 cm.

6. **a.** Draw a straight line segment PQ such that PQ = 10 cm.
- **b.** Mark the point B on the line PQ such that PB = 2 cm.
- **c.** Construct a line perpendicular to PQ through B.
- **d.** Mark a point A on the perpendicular line such that BA = 6 cm and complete the triangle ABQ.

e. Construct the perpendicular bisector of the line segment BQ and name the point it intersects AQ as O.

f. Construct a circle with O as the centre and OA as the radius.

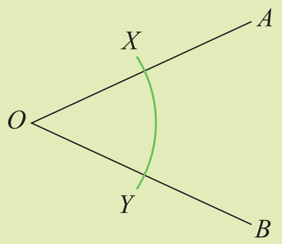
14.4 Constructions related to angles

Constructing the angle bisector

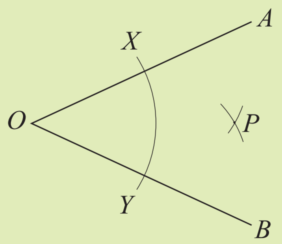
The line drawn through a given angle such that it divides the angle into two equal angles, is known as the angle bisector of the given angle.

Activity 1

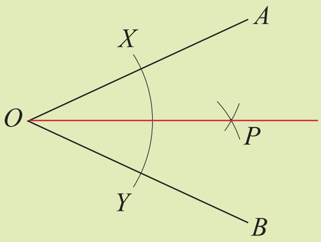
Step 1: Draw an arc with O as the centre such that it intersects the arms OA and OB. Name the points of intersection X and Y.



Step 2 : Using a pair of compasses and taking a suitable radius, construct two arcs with X and Y as the centres such that they intersect each other as shown in the figure. Name the point of intersection P.



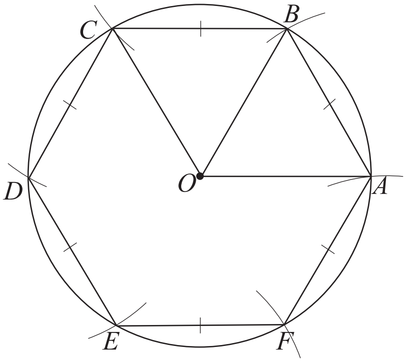
Step 3 : Join OP. Measure $A\hat{O}P$ and $B\hat{O}P$ and check whether they are equal.



It must have been clear to you at the end of the above activity that $A\hat{O}P = B\hat{O}P$. That is, OP is the angle bisector of $A\hat{O}B$.

14.5 Construction of angles

By now we have learnt to draw angles using the protractor. However we can construct a few special angles by using a straight edge and a pair of compasses only. Let us recall how we constructed a regular hexagon in grade 8 by using a pair of compasses.



Here, taking the length of a side of the regular hexagon which needs to be drawn as the radius, a circle is drawn, and with the same radius, arcs are marked on it.The points at which the arcs intersect the circle are joined to each other and to the centre as shown in the figure.

Then every angle of each equilateral triangle that is formed is 60°.

Therefore, $A\hat{O}B = 60^{\circ}$ and $A\hat{O}C = 120^{\circ}$.

Let us use the principles that were used in this construction to construct certain special angles.

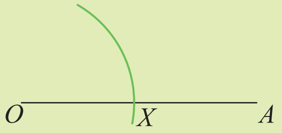
1. Constructing an angle of 60°

Activity 1

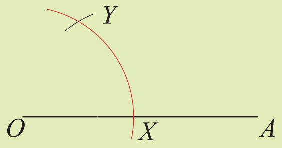
Suppose we need to construct an angle of 60°at O with OA as an arm.

Step 1: Draw a straight line segment in your exercise book and name it OA.

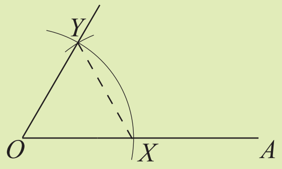
Step 2: Taking O as the centre, construct an arc such that it intersects OA as shown in the figure. Name the point of intersection X.



Step 3: Without changing the length of the radius, and taking X as the centre, draw another arc using the pair of compasses, such that it intersects the first arc. Name this point of intersection Y.



Step 4: Join the points O and Y and produce it as required. Measure $A\hat{O}Y$ and check whether it is 60°.



The triangle OXY in the above figure is an equilateral triangle. The reason for this can be explained as follows.

Since OX and OY are radii of the circle with centre O, $OX = OY$.
Similarly, since XO and XY are radii of the circle with centre X, $XO = XY$.
Accordingly, $OX = XY = OY$.
Therefore, OXY is an equilateral triangle.
Therefore, every angle of it is 60°.

Therefore $X\hat{O}Y = 60^{o}$.

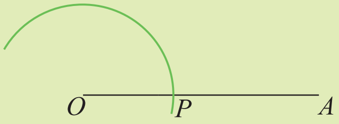
Suppose we need to construct an angle of 120° at O with OA as an arm.

2. Constructing an angle of 120°

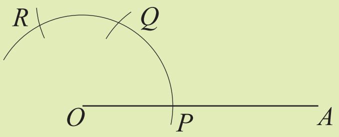
Activity 2

Step 1: Construct a straight line segment and name it OA.

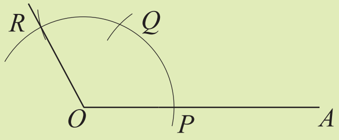
Step 2: Taking O as the centre, construct an arc such that it intersects OA as shown in the figure. Name the point of intersection P.



Step 3: Without changing the length of the radius, and taking P as the centre, draw a small arc using the pair of compasses, such that it intersects the first arc as shown in the figure, and name that point of intersection Q. Now, without changing the radius, take Q as the centre and draw another small arc such that it too intersects the first arc and name that point of intersection R.



Step 4: Join OR and produce it as required. Measure and check the magnitude of $A\hat{O}R$.



The reason why $A\hat{O}R = 120^{o}$ is the following. As discussed above, $P\hat{O}Q = 60^{o}$. Furthermore, QOR is also an equilateral triangle. Therefore, $Q\hat{O}R = 60^{o}$. Accordingly,

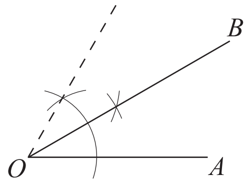
$$\begin{aligned} P\hat{O}R &= P\hat{O}Q + Q\hat{O}R \\ &= 60^{o} + 60^{o} \\ &= 120^{o} \end{aligned}$$

3. Constructing angles of 30°, 90° and 45°

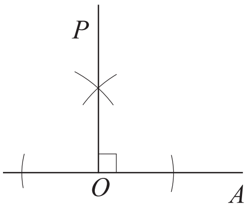
By constructing suitable angle bisectors we can construct the angles 30°, 90° and 45°. By considering the information and figures given below construct the given angles.

Angle of 30°

Construct an angle of 60° and construct its angle bisector. Then $A\hat{O}B = 30^{o}$.

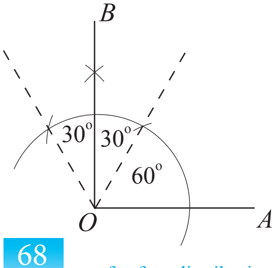


Angle of 90°



Method I

At O, construct a line perpendicular to the line segment AO. Then $A\hat{O}P = 90^{o}$.



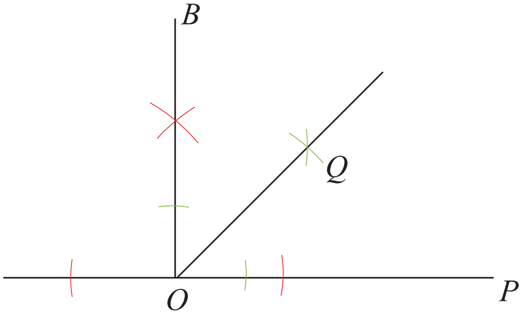
Method II

Construct an angle of 120°and bisect one 60° angle. Then $A\hat{O}B = 90^{o}$.

Angle of 45°

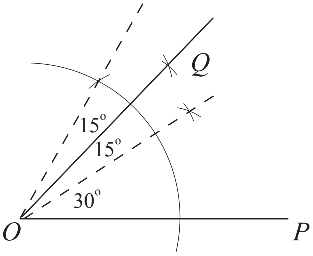
Method I

Construct an angle of 90° and bisect it. Then $P\hat{O}Q = 45^\circ$.



Method II

Construct an angle of 60° and bisect it. Again bisect one of the resulting 30° angles. Then, $P\hat{O}Q = 30^\circ + 15^\circ = 45^\circ$.



Copying a given angle

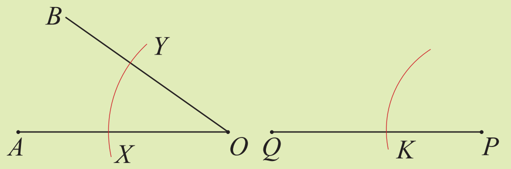
Let us suppose that we need to construct an angle equal to a given angle $A\hat{O}B$ at a point P, with PQ as an arm. For this, let us do the following activity.

Activity 3

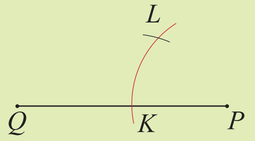
Step 1: Draw any angle and name it $A\hat{O}B$. Draw the arm PQ on which $A\hat{O}B$ needs to be copied.

Step 2: Taking O as the centre, draw an arc as shown in the figure such that it intersects the arms OA and OB, and name the points of intersection X and Y. Using the same radius and taking P as the centre, draw an arc longer than the previous arc such that it intersects PQ.

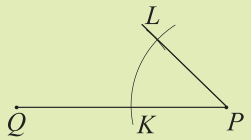
Name the point at which the arc intersects PQ as K.



Step 3: Taking XY as the length of the radius and K as the centre, using the pair of compasses, construct a small arc such that it intersects the initial arc and name the point of intersection L.



Step 4: Join PL and produce it as required. Using a protractor (or any other method), check whether $A\hat{O}B$ and $Q\hat{P}L$ are equal.



Exercise 14.4

1. **i.** Draw a straight line segment of length 8 cm and name it PQ.
   ii. Construct an angle of 60°at P such that PQ is an arm.
   iii. Construct an angle of 60°at Q such that QP is an arm.

2. **i.** Draw a straight line segment of length 6.5 cm and name it AB.
   ii. Construct an angle of 90° at A such that AB is an arm.

iii. Construct an angle of 30°at B such that BA is an arm.

iv. Produce the constructed lines so that they intersect. Name their point of intersection as C and form the triangle ABC.

3. Construct angles of magnitude 15° and 75°.

4. To construct the triangle shown in the figure below, do the following constructions.

i. Draw a straight line segment of length 7 cm and name it PQ.

ii. Construct an angle of 30° at P such that PQ is an arm.

iii. Construct an angle of 45° at Q such that QP is an arm.

iv. Complete the triangle PQR and measure and write the magnitude of $P\hat{R}Q$.

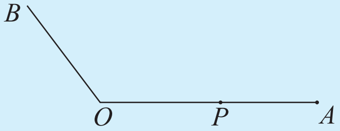
5.

i. Draw a straight line segment OA of length 10 cm.

ii. Draw an arm BO such that $A\hat{O}B$ is an obtuse angle.

iii. Mark the point P on OA such that $OP = 7$ cm.

iv. Construct a line segment PC such that C is on the same side of OA as B and such that $A\hat{P}C = A\hat{O}B$.



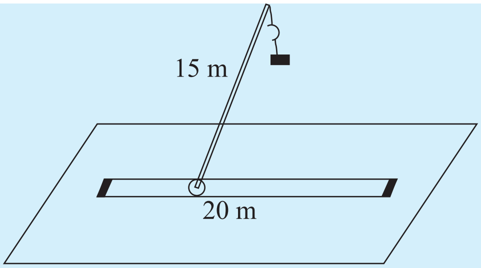
6. **i.** Draw any acute angle and name it $K\hat{L}M$.

ii. Copy the angle $\hat{L}$ at M such that $K\hat{L}M = L\hat{M}N$, where N is on the same side of LM as K.

iii. Name the point of intersection of the lines LK and MN as P (produce the lines if necessary) and measure and write the lengths of PL and PM.

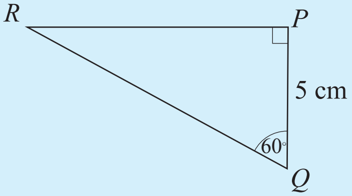
Miscellaneous Exercise

1. In a factory, a 15 m long arm of a crane is fixed to a groove of length 20 m. It can be moved along the groove and also rotated in a horizontal plane about the end points of the groove. Draw a rough sketch, and indicate with measurements the path on the horizontal plane where the crane can exchange goods.

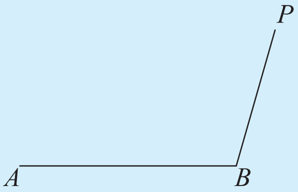


2. To construct the triangle shown in the figure, carry out the steps given below.

i. Draw a straight line segment PQ where $PQ = 5$ cm.
ii. Construct an angle of 90° at P.
iii. Construct an angle of 60° at Q.
iv. Complete the triangle PQR and measure and write the magnitude of $\hat{R}$.

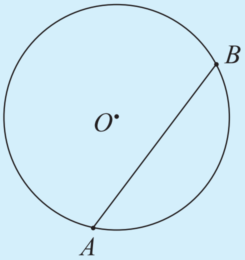


3. i. As shown in the figure, draw an obtuse angle $A\hat{B}P$.



ii. Locate a point K such that $A\hat{B}P = B\hat{P}K$ and such that the two angles form a pair of alternate angles. Join PK.

4. **i.** Draw a circle of radius 4 cm and name its centre O.
ii. Mark two points A and B on the circle 6 cm apart from each other, and draw the line AB.
iii. Construct a perpendicular line from O to AB and name the point at which it meets AB as N.
iv. Measure and write the lengths of AN and BN.



Summary

A set of points satisfying one or more conditions is known as a locus.

Basic Loci

- **The locus of points on a plane which are at a constant distance from a fixed point is a circle.**
- **The locus of points which are equidistant from two given points is the perpendicular bisector of the line joining the two points.**
- **The locus of points which are at a constant distance from a straight line are the two straight lines parallel to it and at the given constant distance from it, on either side of it.**
- **The locus of points equidistant from two intersecting straight lines is the angle bisector of the angles formed by the intersection of the two lines.**

15

Equations

By studying this lesson you will be able to;

- solve linear equations containing brackets,
- solve linear equations containing fractions,
- solve simultaneous linear equations when the coefficient of one unknown is equal in both equations.

Linear equations

Do the following exercise to recall the facts that you have learnt in previous grades on solving linear equations.

Review Exercise

Solve the following linear equations.

a. $x + 12 = 20$
b. $x - 7 = 2$
c. $5 + m = 8$
d. $2x = 16$
e. $-3x = 6$
f. $2p + 1 = 5$
g. $3b - 7 = 2$
h. $\frac{x}{2} = 3$
i. $\frac{2p}{2} = 6$
j. $\frac{m}{5} - 1 = 8$
k. $2(x + 3) = 11$
l. $3(1 - x) = 9$

15.1 Solving linear equations with two types of brackets

You may have observed that there are some equations with brackets in the review exercise. In this lesson we expect to learn how to solve linear equations with two types of brackets. Let us first consider how to construct a linear equation with several brackets and find its solution.

Note: There are several types of brackets that we use.

( )	{ }	[ ]
↑	↑	↑
Parentheses	Curly Brackets	Square Brackets

When applying brackets, the usual practice is to first use parentheses, then curly brackets and finally square brackets.

"The result of adding three to a certain number and subtracting one from twice this value, and finally multiplying the resulting value by five and adding two is equal to 47".
Let us consider how to construct an equation using the above information and then solve it.

If the number is x, when 3 is added, we obtain $x + 3$.

Twice this expression can be written as $2(x + 3)$ using parentheses.

The expression that is obtained when 1 is subtracted from this is $2(x+3) -1$.

Using curly brackets to write five times this expression we obtain,

$$5\{2\,(x + 3) - 1\}$$

It is given that when 2 is added to this expression it is equal to 47. Therefore,

$$5\{2\,(x + 3) - 1\} + 2 = 47$$

Now, by solving this equation, let us find the value of the number (x).

First, by simplifying the expression with parentheses we obtain

$$5\{2x + 6 - 1\}+ 2 = 47.$$

When we simplify the expression within curly brackets we obtain,

$$5\{2x + 5\} + 2 = 47.$$

Now, simplifying the expression with curly brackets we obtain

$$\begin{aligned} 10x + 25 + 2 &= 47. \\ 10x + 27 \quad &= 47 \end{aligned}$$

Subtracting 27 from both sides we obtain,

$$10x + 27 - 27 = 47 - 27.$$

That is, $10x = 20$.

Dividing both sides by10 we obtain,

$$\frac{10x}{10} = \frac{20}{10}$$

$$x = 2$$

Therefore, the number is 2.

Let us consider a few more examples of equations with brackets to improve our skills of solving such equations.

Example 1

Solve $2\{3\,(2x-1)+4\}=38$.

$$\frac{2\{3\,(2x-1)+4\}}{2}=\frac{38}{2}\quad\text{(dividing both sides by 2)}$$

$$3(2x-1)+4=19$$

$$6x-3+4\;=19\ \text{(simplifying the expression with parentheses)}$$

$$6x+1=19$$

$$6x+1-1=19-1\ \text{(subtracting 1 from both sides)}$$

$$6x=18$$

$$\frac{6x}{6}=\frac{18}{6}\quad\text{(dividing both sides by 6)}$$

$$\underline{\underline{x=3}}$$

Example 2

Solve $5\{4\,(x+3)-2\,(x-1)\}=72$.

$$5\{4\,(x+3)-2\,(x-1)\}=72$$

$$5\{4x+12-2x+2\}=72\quad\text{(simplifying the expression with parentheses)}$$

$$5\{2x+14\}=72$$

$$10x+70=72\quad\text{(simplifying the expression with curly brackets)}$$

$$10x+70-70=72-70\ \text{(subtracting 70 from both sides)}$$

$$\frac{10x}{10}=\frac{2}{10}\quad\text{(dividing both sides by 10)}$$

$$\underline{\underline{x=\frac{1}{5}}}$$

Exercise 15.1

Solve the following equations.

a. $2\{2\,(x-1)+2\}=18$

b. $5\{3\,(x+2)-2\,(x-1)\}=60$

c. $6+2\,\{x+3\,(x+2)\}=58$

d. $5\{2+3\,(x+2)\}=10$

e. $2\,\{3\,(y-1)-2y\}=2$

f. $7x+5\,\{4-(x+1)\}=17$

15.2 Solving linear equations containing fractions

Now, let us consider how to construct a linear equation with fractions and find its solution.

A vendor bought a stock of mangoes to sell. He discarded 10 fruits which were rotten. The rest he divided into 12 equal piles of 5 mangoes each.

Let us construct an equation using the above data.

If the vender bought x mangoes to sell,
when 10 mangoes are discarded, the remaining amount is $x - 10$.
The number of piles that can be made when the remaining mangoes are divided into groups of 5 is $\frac{x-10}{5}$. It is given that the number of piles is 12.

Therefore, $\frac{x-10}{5} = 12$

Now, let us solve this equation and find x.

$$\frac{x-10}{5} = 12$$

Multiplying both sides of the equation by 5,

$$5 \times \frac{x-10}{5} = 12 \times 5$$

$$x - 10 = 60$$

Adding 10 to both sides,

$$x - 10 + 10 = 60 + 10$$

$$x = 70$$

Therefore, the vendor bought 70 mangoes to sell.

Let us study the following examples to learn more on solving linear equations with fractions.

Example 1

Solve $\frac{x+3}{2} = 15$.

$$\frac{x+3}{2} = 15$$

$$2 \times \frac{x+3}{2} = 15 \times 2 \text{ (multiplying both sides by 2)}$$

$$x + 3 = 30$$

$$x + 3 - 3 = 30 - 3 \text{ (subtracting 6 from both sides)}$$
$$\underline{\underline{x = 27}}$$

Example 2

Solve $\frac{y}{2} - \frac{y}{3} = 9$.

$$\frac{y}{2} - \frac{y}{3} = 9$$
$$6 \times \frac{y}{2} - 6 \times \frac{y}{3} = 9 \times 6 \text{ (multiplying both sides by 6, the L.C.M. of the denominators 2 and 3)}$$
$$3y - 2y = 54$$
$$\underline{\underline{y = 54}}$$

Example 3

Solve $2\left(\frac{m}{3} - 1\right) = 10$.

$$2\left(\frac{m}{3} - 1\right) = 10$$
$$\frac{2}{2}\left(\frac{m}{3} - 1\right) = \frac{10}{2} \text{ (dividing both sides by 2)}$$
$$\frac{m}{3} - 1 = 5$$
$$\frac{m}{3} - 1 + 1 = 5 + 1 \text{ (adding 1 to both sides)}$$
$$\frac{m}{3} = 6$$
$$3 \times \frac{m}{3} = 6 \times 3 \text{ (multiplying both sides by 3)}$$
$$\underline{\underline{m = 18}}$$

Note: When solving equations, it is not necessary to write the reason for each simplification.

Exercise 15.2

Solve each of the following equations.

a. $\frac{x - 2}{5} = 4$

b. $\frac{y + 8}{3} = 5$

c. $\frac{2a}{3} + 1 = 7$

d. $\frac{5b}{2} - 3 = 2$

e. $\frac{2p + 3}{4} = 5$

f. $\frac{3m - 2}{7} = 4$

g. $\dfrac{3x}{2} + \dfrac{x}{4} = 7$

h. $\dfrac{2m}{3} - \dfrac{3m}{5} = 1$

i. $4\left(\dfrac{3x}{2} - 1\right) = 12$

j. $\dfrac{1}{3}\left(\dfrac{2a}{3} - 3\right) = 2$

k. $\dfrac{m-3}{2} + 1 = 4$

l. $\dfrac{x+1}{2} + \dfrac{x}{3} = 8$

m. $\dfrac{y+1}{2} + \dfrac{y-3}{4} = \dfrac{1}{2}$

n. $\dfrac{x+3}{2} - \dfrac{x+1}{3} = 2$

15.3 Solving simultaneous equations

You have learnt in previous grades and in the earlier section of this lesson how to find the value of the unknown by solving a linear equation.
In this section we will learn how to solve linear equations with two unknowns.

Suppose it is given that the sum of two numbers is 6.

If we take the two numbers as x and y, then we can construct the equation $x + y = 6$, based on the given statement.
Here, x and y are not unique. The following table shows several different pairs of values of x and y which satisfy the above equation.

x	y	$x + y$
– 1	7	6
0	6	6
1	5	6
2	4	6
3	3	6
4	2	6
5	1	6
6	0	6

Table 1

By observing the above table, we can conclude that there are infinitely many pairs of values of x and y which satisfy the equation $x + y = 6$.
If there is another relationship between x and y, we can construct another equation and by solving both equations simultaneously we can find the values of x and y that satisfy both equations.

Suppose it is given that the difference of the two numbers is 2. If we take the larger number as x, we can construct the equation $x - y = 2$, based on the given statement.

There are infinitely many pairs of values of x and y which satisfy this equation too as can be concluded from observing the following table.

x	y	$x - y$
6	4	2
5	3	2
4	2	2
3	1	2
2	0	2
1	–1	2

Table 2

By observing Tables 1 and 2, you can see that there is only one pair of values of x and y which satisfies both $x + y = 6$ and $x - y = 2$. This pair is $x = 4$ and $y = 2$. Therefore, the solution of the above two equations is $x = 4$ and $y = 2$.

A pair of equations of this type with two unknowns is known as a pair of simultaneous equations. “Simultaneous” means “occurring at the same time”.
Let us learn how to solve pairs of simultaneous equations using several other methods which are shorter, by considering the following examples.

Example 1

Solve the pair of simultaneous equations $x + y = 6$ and $x - y = 2$.

To facilitate finding the solution, let us label the two equations as ① and ② .

$x + y = 6$ _______ ①

$x - y = 2$ _______ ②

Method I

We can name this method “the method of substitution”.

By making x the subject of equation ② , we can write it as

$$x = 2 + y.$$

By substituting this expression for x in equation ① we obtain,

$$2 + y + y = 6.$$
$$2 + 2y = 6$$

This is a linear equation in one unknown.

Let us find the value of y by solving it.

$$2 - 2 + 2y = 6 - 2$$
$$2y = 4$$
$$\frac{2y}{2} = \frac{4}{2}$$
$$\underline{\underline{y = 2}}$$

We can now find the value of x by substituing $y = 2$ in $x = 2 + y$.

$$x = 2 + 2$$
$$\underline{\underline{x = 4}}$$

Method II

This method can be named "the method of elimination".

$x + y = 6$ _______ ①
$x - y = 2$ _______ ②

First, observe that $+ y$ occurs in equation (1) and $- y$ occurs in equation ②.
By adding both equations we get
$x + y + x - y = 6 + 2$

Here we have used the axiom "Quantities which are obtained by adding equal quantities to equal quantities, are equal".

Now we obtain a linear equation in x, since $+ y$ and $-y$ cancel off.
Let us solve it and find the value of x.

$$2x = 8$$
$$\frac{2x}{2} = \frac{8}{2}$$
$$\underline{\underline{x = 4}}$$

To find the value of y, let us substitute $x = 4$ in equation ①,

$$4 + y = 6$$
$$4 - 4 + y = 6 - 4$$
$$\underline{\underline{y = 2}}$$

$x = 4$
$y = 2$

Note that in the above pair of simultaneous equations, the coefficient of y was 1 in one equation and -1 in the other. That is, the numerical values of these coefficients are equal (when the signs are ignored).

Let us consider a few more examples. We will use the 2nd method to solve them.

Example 2

Solve $2m + n = 10$

$m - n = 2$.

$2m + n = 10 \longrightarrow$ ①

$m - n = 2 \longrightarrow$ ②

Adding ① and ②, $2m + n + m - n = 10 + 2$

$$3m = 12$$

$$\frac{3m}{3} = \frac{12}{3}$$

$$\underline{\underline{m = 4}}$$

By substituting $m = 4$ in ①,

$$2 \times 4 + n = 10$$

$$8 + n = 10$$

$$n = 10 - 8$$

$$\underline{\underline{n = 2}}$$

$m = 4$

$n = 2$

Example 3

Solve $2a + b = 7$

$a + b = 4$.

$2a + b = 7 \longrightarrow$ ①

$a + b = 4 \longrightarrow$ ②

In these equations, the coefficient of b is equal. Therefore, to eliminate b, we must subtract one equation from the other.

① – ②, $2a + b - (a + b) = 7 - 4$ (As there is a subtraction, it is essential to use brackets and write $(a + b)$)

$$2a + b - a - b = 3$$

$$\underline{\underline{a = 3}}$$

By substituting $a = 3$ in ②,

$$3 + b = 4$$

$$b = 4 - 3$$

$$\underline{\underline{b = 1}}$$

Example 4

Solve $x + 2y = 11$
$x - 4y = 5$.

$x + 2y = 11 \longrightarrow$ ①
$x - 4y = 5 \longrightarrow$ ②

Here the coefficients of x are equal. Therefore, let us subtract one equation from the other to eliminate x.

①$-$②, $x + 2y - (x - 4y) = 11 - 5$

$$x + 2y - x + 4y = 6$$
$$6y = 6$$
$$\frac{6y}{6} = \frac{6}{6}$$
$$\underline{\underline{y = 1}}$$

By substituting $y = 1$ in ① ,

$$x + 2 \times 1 = 11$$
$$x + 2 = 11$$
$$x + 2 - 2 = 11 - 2$$
$$\underline{\underline{x = 9}}$$

Exercise 15.3

1. Solve each of the following pairs of simultaneous equations.

a. $a + b = 5$
$a - b = 1$

b. $x + y = 8$
$2x + y = 2$

c. $m + 2n = 7$
$m - n = 1$

d. $4c - b = 7$
$4c - 2b = 2$

e. $2a + 3b = 16$
$4a + 3b = 26$

f. $3k + 4l = 4$
$3k - 2l = 16$

g. $x + 3y = 12$
$-x + y = 8$

h. $3m - 2n = 10$
$-3m + n = -14$

2. The sum of two numbers is 10 and their difference is 2. Taking the two numbers as x and y, construct a pair of simultaneous equations and solve them.

3. Two pens and a pencil cost Rs 32. A pen and a pencil cost Rs 20. Taking the price of a pen as Rs p and the price of a pencil as Rs q,construct a pair of simultaneous equations and by solving the pair find the price of a pen and the price of a pencil.

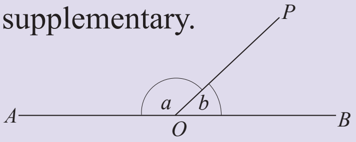
16 Angles of a Triangle

By studying this lesson you will be able to;

- solve simple problems using the theorem "The sum of the interior angles of a triangle is 180°",
- solve simple problems using the theorem "The exterior angle of a triangle is equal to the sum of the interior opposite angles".

Let us recall several results in geometry that you have learnt earlier related to straight lines.

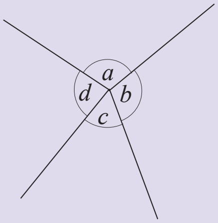
- A pair of adjacent angles on a straight line are supplementary.



AOB is a straight line.

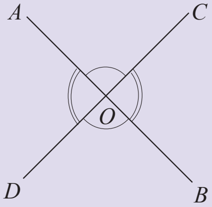
$\therefore\ a + b = 180^{\circ}$.

- The sum of the angles around a point is 360°.



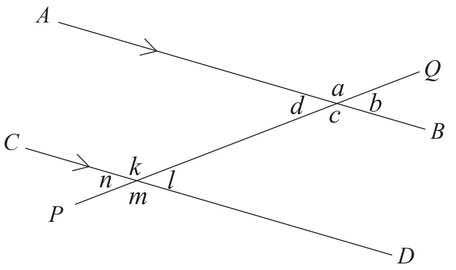
$$a + b + c + d = 360^{\circ}$$

- The vertically opposite angles formed by the intersection of two straight lines are equal.



AB and CD are straight lines. $A\hat{O}C = B\hat{O}D$ and $A\hat{O}D = C\hat{O}B$.

- Angles related to parallel lines

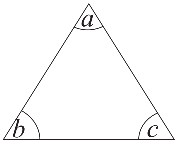


$AB \parallel CD$.

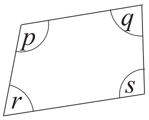
- $c = k$ and $d = l$ (alternate angles)
- $a = k,\ b = l,\ d = n,\ c = m$ (corresponding angles)
- $d + k = 180^{\circ}$ and $c + l = 180^{\circ}$ (allied angles)

In the lesson on triangles and quadrilaterals learnt in grade 8, we identified that;

- the sum of the interior angles of a triangle is 180° and the sum of the interior angles of a quadrilateral is also 360°.

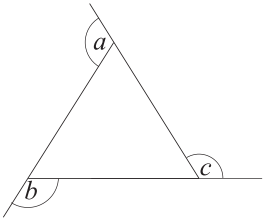


$a + b + c = 180^{\circ}$

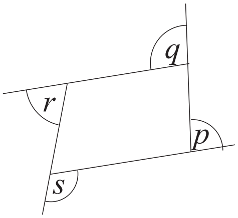


$p + q + r + s = 360^{\circ}$

- the sum of the exterior angles of a triangle is 180° and the sum of the exterior angles of a quadrilateral is also 360°.



$a + b + c = 360^{\circ}$



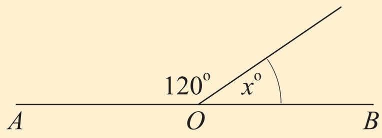
$p + q + r + s = 360^{\circ}$

Do the following review exercise to further establish the above facts.

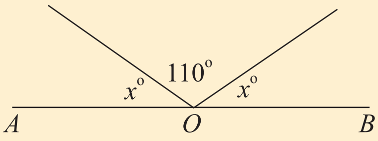
Review Exercise

a. *AOB* is a straight line. Find the value of x.

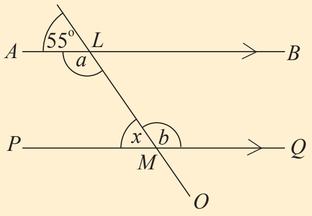
i.



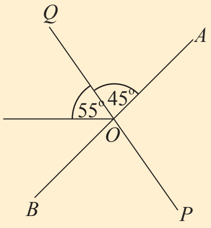
ii.



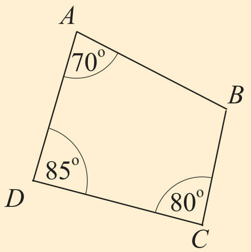
b. Find the magnitude of each of the angles a, b and x, using the information in the figure.



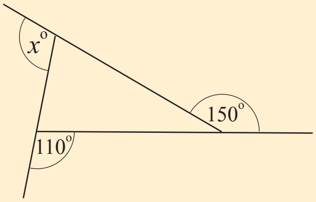
c. *AOB* and *POQ* are straight lines. Find the magnitudes of $P\hat{O}B$, $Q\hat{O}B$ and $A\hat{O}P$.



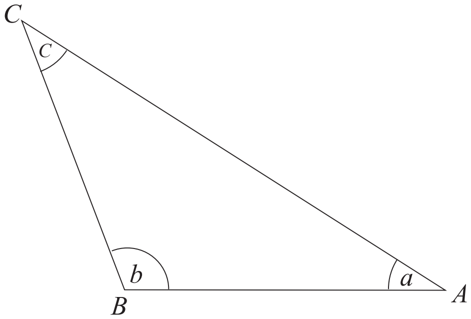
d. Find the magnitude of $A\hat{B}C$ using the information in the figure.



e. Find the value of x, using the information in the figure.



16.1 Interior angles of a triangle



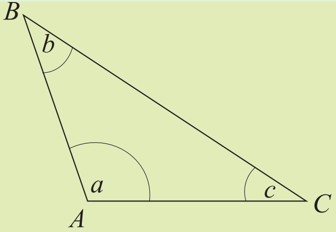
a, b and c are the interior angles of the triangle ABC in the above figure. As discussed earlier, the sum of the interior angles of a triangle is 180°. Hence,

$$A\hat{B}C + B\hat{C}A + C\hat{A}B = 180^\circ.$$

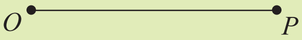
Let us do the following activity to verify the above relationship.

Activity 1

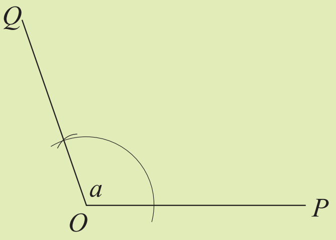
Step 1: Draw a triangle in your exercise book and name it ABC. (The interior angles are given as a, b, c).



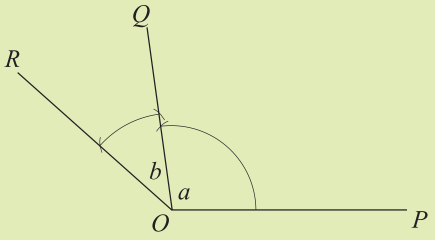
Step 2: Draw a straight line segment in your exercise book and name it OP.



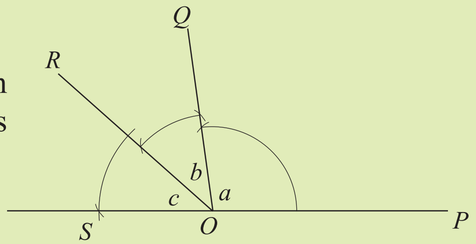
Step 3: Using a straight edge and a pair of compasses, copy the angle $C\hat{A}B$ (a) at O, such that O is the vertex and OP is an arm. (This angle is indicated as $P\hat{O}Q$ in the figure).



Step 4: As done above, copy the angle $A\hat{B}C$ at O, such that O is the vertex and OQ is an arm. (This angle is indicated as $Q\hat{O}R$ in the figure).



Step 5: Copy the angle $A\hat{C}B$ at O, such that O is the vertex and OR is an arm. (This angle is indicated as $R\hat{O}S$ in the figure).



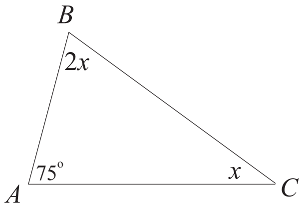
Examine whether *POS* is a straight line by using a straight edge or a protractor, and accordingly write the conclusion we can arrive at.

In the above activity, you must have established that *POS* is a straight line. Since the interior angles of the triangle *ABC* were copied onto the straight line *POS*, and since the sum of the angles on a straight line is 180°, it can be concluded that the sum of the interior angles of the triangle *ABC* is 180°. This can be stated as a theorem as follows.

Theorem: **The sum of the three interior angles of a triangle is** 180°.

Now let us consider a few examples to see how this theorem can be used to solve problems.

Example 1



Determine the magnitudes of $A\hat{C}B$ and $A\hat{B}C$ of the triangle ABC, using the information given in the figure.

$$\begin{aligned}
75^{\circ} + 2x + x &= 180^{\circ} \\
3x &= 180^{\circ} - 75^{\circ} \\
3x &= 105^{\circ} \\
x &= \frac{105^{\circ}}{3} \\
&= 35^{\circ}
\end{aligned}$$

$$\therefore\ A\hat{C}B = x = 35^{\circ}$$

$$A\hat{B}C = 2x = 2 \times 35^{\circ} = 70^{\circ}$$

Example 2

The magnitudes of the interior angles of a triangle are in the ratio 2:3:4. Determine the magnitudes of these angles and giving reasons mention what type of triangle it is.

The ratio of the magnitudes of the angles = 2: 3: 4

$\therefore$ The fractions related to the angles = $\frac{2}{9}, \frac{3}{9}, \frac{4}{9}$

The sum of the three angles = 180°

$\therefore$ The smallest angle = $\overset{20}{\cancel{180}}{}^{\circ} \times \frac{2}{\underset{1}{\cancel{9}}} = 40^{\circ}$

The medium angle = $180^{\circ} \times \frac{3}{9} = 60^{\circ}$

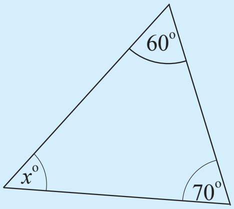
The largest angle = $180^{\circ} \times \frac{4}{9} = 80^{\circ}$

Hence, the interior angles of the triangle are of magnitudes 40°, 60° and 80°. This is an acute triangle since every angle is less than 90°.

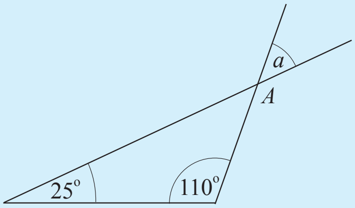
Exercise 16.1

1. Find the magnitude of each angle indicated by a lowercase letter in the following figures, using the information provided in the figures.

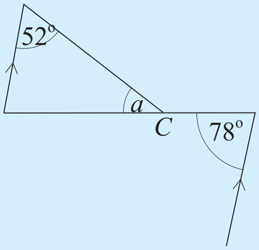
i.



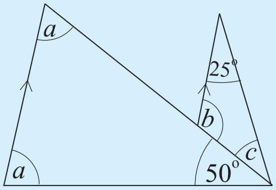
ii.



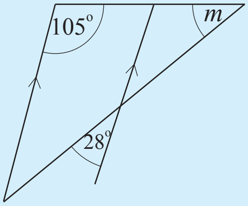
iii.



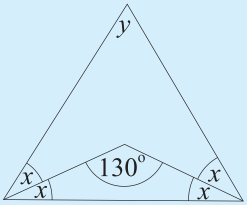
iv.



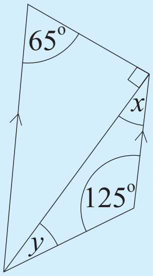
v.



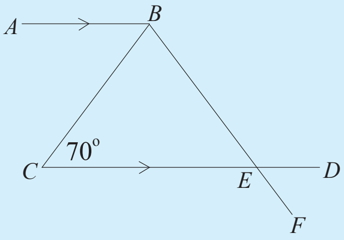
vi.



vii.

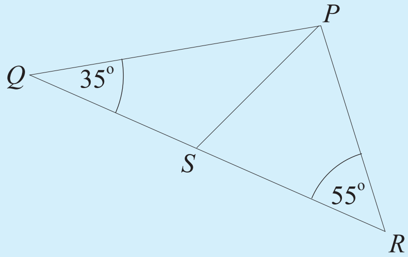


2.



In the given figure, $A\hat{B}C = C\hat{B}E$. $B\hat{C}E = 70^{\circ}$. Find the magnitude of $D\hat{E}F$.

3.



In the triangle PQR, the point S is located on QR such that $Q\hat{P}S = R\hat{P}S$. Moreover, $P\hat{Q}S = 35^\circ$ and $P\hat{R}S = 55^\circ$.

(i) Find the magnitude of $Q\hat{P}R$.

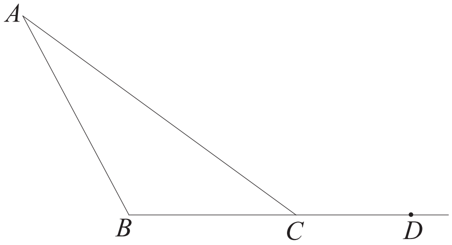
(ii) Find the magnitude of $P\hat{S}R$.

4. In the triangle XYZ, $\hat{X} + \hat{Y} = 115^\circ$ and $\hat{Y} + \hat{Z} = 100^\circ$. Find the magnitudes of $\hat{X}$, $\hat{Y}$ and $\hat{Z}$.

5. The ratio of the magnitudes of the interior angles of a triangle is 1: 2 : 3. Find the magnitude of each angle separately and with reasons mention what type of a triangle it is.

6. An interior angle of a triangle is 75°. The ratio of the magnitudes of the remaining two angles is 1 : 2. Find the magnitude of each of these angles.

16.2 Exterior angles of a triangle

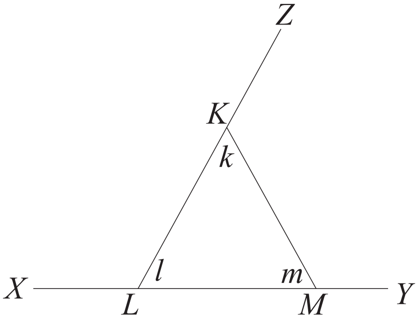


The side BC of the triangle ABC shown in the figure is produced and the point D is marked on BC produced. The angle $A\hat{C}D$ which is formed outside the triangle is called an exterior angle of the triangle.

The interior angle of the triangle, which is adjacent to the exterior angle $A\hat{C}D$ is $A\hat{C}B$. The other two interior angles which are not adjacent to the exterior angle are called the interior opposite angles.

Accordingly, in this figure, the interior opposite angles relevant to the exterior angle $A\hat{C}D$ are $C\hat{A}B$ and $A\hat{B}C$.

Now, let us consider another instance.



In the triangle *KLM* in the above figure, *k*, *l* and *m* are the interior angles. Three exterior angles have been created by producing the sides of the triangle.

The interior opposite angles relevant to $K\hat{M}Y$ are k and l.

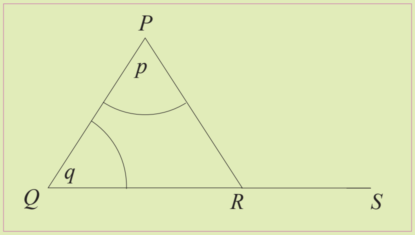
The interior opposite angles relevant to $M\hat{K}Z$ are l and m.

The interior opposite angles relevant to $X\hat{L}K$ are k and m.

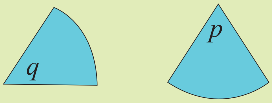
Now let us develop a relationship between an exterior angle and the interior opposite angles of a triangle.

Activity 1

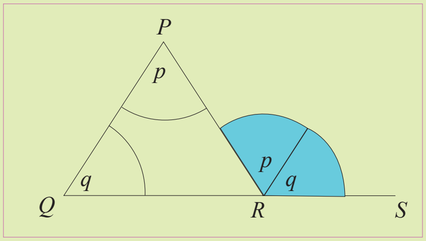
Step 1: Draw a triangle on a piece of Bristol board or on a thick sheet of paper as shown in the figure. Produce a side to create an exterior angle. Mark and shade the interior opposite angles relevant to it (Indicated by *p* and *q* in the figure).



Step 2: Using a blade, cut and separate out the interior opposite angles you marked as laminas.



Step 3: Place the two laminas of the interior opposite angles such that they coincide with the exterior angle as shown in the figure, and paste them

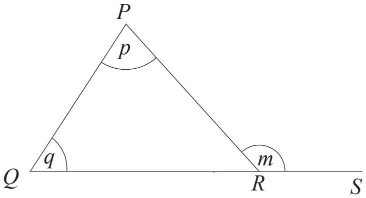


Compare your completed work with those of your friends. Write the conclusion that can be arrived at through this activity.

From the above activity, we can say that the exterior angle of a triangle is equal to the sum of the interior opposite angles.

Draw an acute triangle, a right triangle and an obtuse triangle in your exercise book and in each triangle, mark an exterior angle and the relevant interior opposite angles. Measure them using a protractor and verify the above relationship for the three triangles by obtaining the sum of the interior opposite angles.

The above result can be expressed as follows.



$m = p + q$.

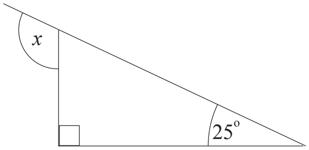
That is, $P\hat{R}S = R\hat{P}Q + P\hat{Q}R$.

This can be expressed as a theorem.

Theorem: **If a side of a triangle is produced, the exterior angle so formed is equal to the sum of the two interior opposite angles.**

Now, let us consider a few examples to see how this result can be used to solve problems.

Example 1



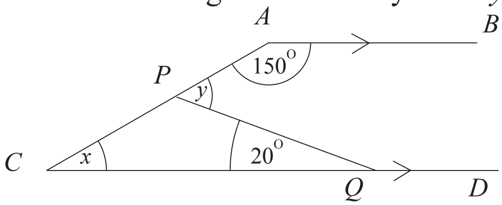
Find the magnitude of the angle indicated by x in the figure.

$x = 90^\circ + 25^\circ$

$= \underline{\underline{115^\circ}}$

Example 2

Find the magnitudes of the angles denoted by x and y in the figure.



$x + 150^\circ = 180^\circ$ (since $AB \parallel CD$ and allied angles are supplementary)

$x = 180^\circ - 150^\circ = 30^\circ$

$y = x + 20^\circ$ (the exterior angle of triangle PCQ = the sum of the interior opposite angles)

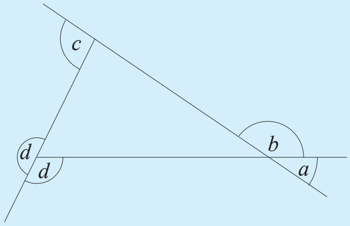
$y = 30^\circ + 20^\circ$

$= \underline{\underline{50^\circ}}$

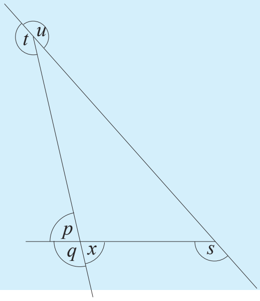
Exercise 16.2

1. Select and write the letters corresponding to the angles which are exterior angles of the given triangles.

i.

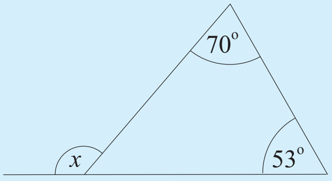


ii.

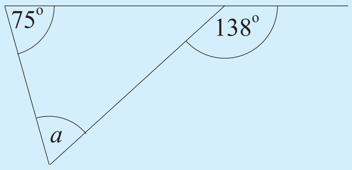


2. Find the magnitude of each angle denoted by a lowercase letter in the following figures.

i.



ii.



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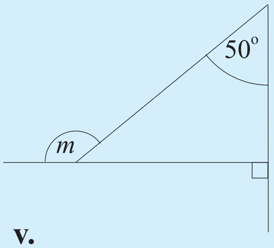

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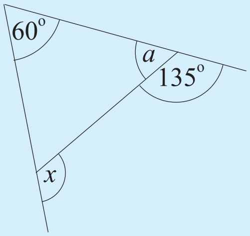
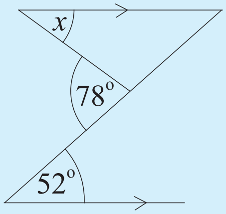
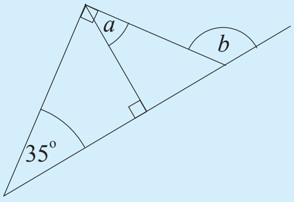


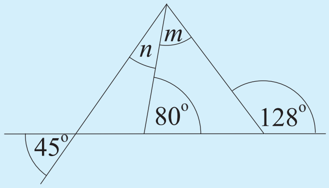
v.



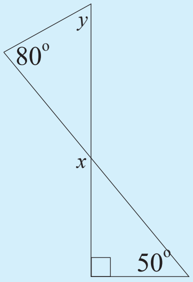
vi.



vii.

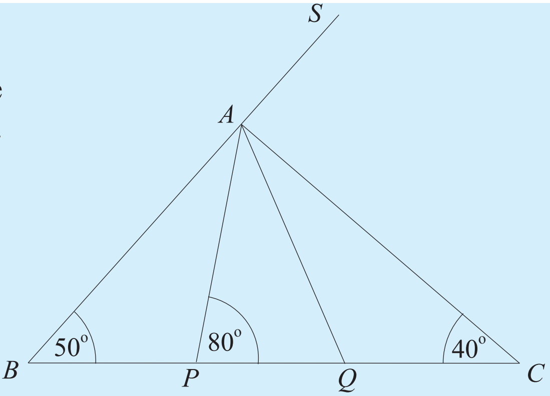


viii.

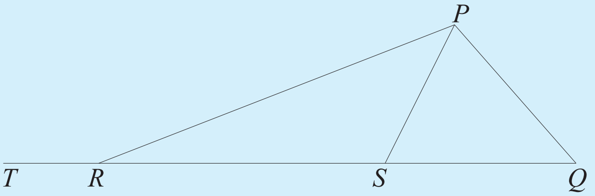


3. In the triangle ABC in the figure, the points P and Q are located on the side BC such that $B\hat{A}P = C\hat{A}Q$. The side BA is produced to S.

i. Find the magnitude of $B\hat{A}P$.
ii. Find the magnitude of $A\hat{Q}P$.
iii. Find the magnitude of $S\hat{A}Q$.

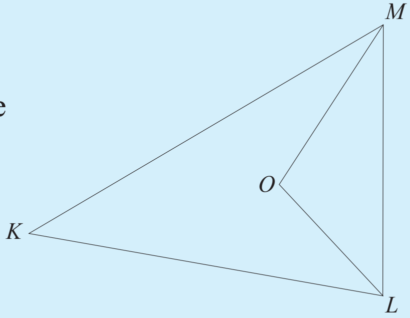


4. In the triangle PQR shown in the figure, the bisector of $\hat{P}$ meets QR at S. Moreover, $S\hat{P}Q = S\hat{Q}P$. If $S\hat{Q}P = a^{\circ}$, then find $P\hat{R}T$ in terms of a.

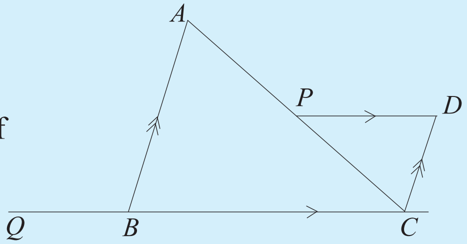


Miscellaneous Exercise

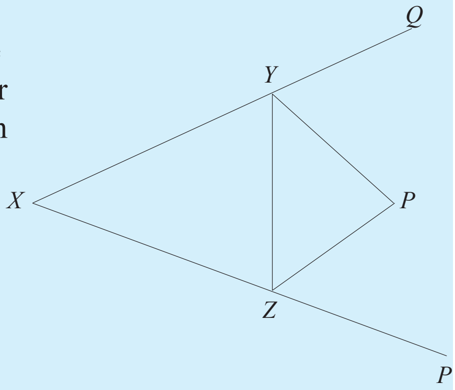
1. In the triangle KLM, the angle bisectors of $\hat{M}$ and $\hat{L}$ meet at O. Moreover, $\hat{K} = 70^{\circ}$. Find the magnitude of $L\hat{O}M$.



2. In the given figure, $A\hat{P}D = 140^{\circ}$ and $P\hat{D}C = 85^{\circ}$. Find the magnitude of $A\hat{B}Q$.



3. The sides XY and XZ of the triangle XYZ have been produced. The bisectors of the exterior angles at Y and Z intersect at P. Find $Y\hat{P}Z$ in terms of $\hat{X}$.



Summary

- **The sum of the three interior angles of a triangle is 180°.**
- **If a side of a triangle is produced the exterior angle so formed is equal to the sum of the two interior opposite angles.**

17

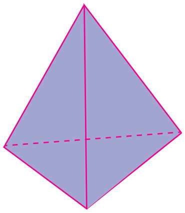
Formulae

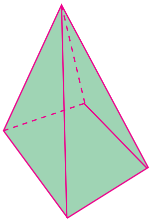
By studying this lesson you will be able to;

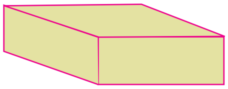
- change the subject of a formula,
- find the value of a variable term in a formula when the values of the other variables are given.

Introducing formulae

In grade 8 you learnt Euler's relationship which is an equation expressing the relationship between the number of edges, number of vertices and number of faces in a solid.







This relationship is the following.

$$\text{Number of edges} = \text{Number of vertices} + \text{Number of faces} - 2$$

By taking the number of edges as E, the number of vertices as V and the number of faces as F, this relationship can be expressed as follows.

$$E = V + F - 2$$

A relationship between two or more quantities expressed as an equation, is known as a "formula".

The quantities in a formula are known as variables. In general, a formula is written with just one variable which is called the "subject of the formula" on one side of the equal sign (usually the left hand side), and the remaining variables on the other side. For example, in the formula $E = V + F - 2$ stated above, E is the subject.

Let us consider another formula. Temperature can be expressed in degrees Celsius or degrees Fahrenheit. The relationship between these two units is given below.

$$F = \frac{9}{5}C + 32$$

Here, F denotes the temperature in Fahrenheit and C denotes the temperature in Celsius. The subject of this formula is F.

Some formulae that are frequently used in Science and Mathematics are given below.

$p = 2\,(a + b)$

$v = u + at$

$s = \frac{n}{2}\,(a + l)$

$y = mx + c$

$C = 2\,\pi r$

$A = \pi r^2$

17.1 Changing the subject of a formula

E is the subject of the formula $E = V + F - 2$. If required, we can make either V or F the subject of this formula. This can be done in a manner similar to solving equations by using axioms.

As an example, let us make V the subject of the formula $E = V + F - 2$.
V is on the right hand side of this equation. F and -2 are also on the same side of the equation as V. To remove the terms F and -2 from the right hand side, let us add $-F$ and $+2$ to both sides of the equation.

We then obtain, $E + (-F) + 2 = V + F - 2 + (-F) + 2$.

Now, by simplifying both sides we obtain,
$E - F + 2 = V$ (since $F + (-F) = 0$ and $-2 + 2 = 0$)

The subject V appears on the right hand side.
Since the subject is usually written on the left hand side, we re-write the above equation as follows with V on the left hand side.
$V = E - F + 2$

The following examples show how the subject of formulae of various forms are changed.

Example 1

Make a the subject of the formula $v = u + at$.

Here the variable a is multiplied by the variable t. Therefore, we need to first make the term at the subject.
Subtracting u from both sides of $v = u + at$ we obtain

$$v - u = u + at - u$$
$$v - u = at$$

Now by dividing both sides by t to make a the subject we obtain,

$$\frac{v-u}{t} = \frac{at}{t}$$

By simplifying this we get the formula $a = \dfrac{v-u}{t}$ with a as the subject.

Example 2

Make n the subject of the formula $S = \dfrac{n}{2}\,(a + l)$.

$$S = \frac{n}{2}\,(a + l).$$

Here, the variable n which is to be made the subject is divided by 2 and the result is multiplied by $(a + l)$. Therefore both sides of the formula need to be multiplied by 2 and divided by $(a + l)$ to make n the subject.

By multiplying both sides by 2 we obtain,

$$2S = \cancel{2}^{1} \times \frac{n}{\cancel{2}_{1}} \times (a + l)$$
$$2S = n(a + l)$$

Now, by dividing both sides by $(a + l)$ we obtain

$$\frac{2S}{a+l} = \frac{n\,\cancel{(a+l)}^{1}}{\cancel{(a+l)}_{1}}$$

$$\frac{2S}{a+l} = n$$

$$n = \frac{2S}{a+l}$$

Example 3

Make n the subject of the formulal $l = a + (n - 1)d$.

$l = a + (n - 1)d$

Let us consider the variable n which is to be made the subject. Observe that the right hand side of the formula is formed by subtracting 1 from n to obtain $(n - 1)$, then multiplying $(n - 1)$ by d to obtain $(n - 1)d$ and finally adding a to $(n - 1)d$.

To make n the subject, we need to perform the inverse operations corresponding to the arithmetic operations performed in the above three steps (i.e., the inverse operation “addition” of the operation “subtraction, the inverse operation “division” of the operation “multiplication”, etc.), starting from the last step and moving up.

Expressed in another way, this means that we make n the subject of the formula by using the relevant axioms.

Therefore, let us first subtract a from both sides of the equation and simplify.

$$l = a + (n - 1)d$$
$$l - a = a + (n - 1)d - a$$
$$l - a = (n - 1)d$$

Now let us divide both sides by d and simplify.

$$\frac{l-a}{d} = \frac{(n-1)\cancel{d}^{1}}{\cancel{d}_{1}}$$
$$\therefore \quad \frac{l-a}{d} = n - 1$$

Finally let us add 1 to both sides and simplify.

$$\frac{l-a}{d} + 1 = n - 1 + 1$$
$$\frac{l-a}{d} + 1 = n$$
$$n = \frac{l-a}{d} + 1$$

If required, you may simplify the right hand side further, using a common denominator. However it is not essential to do this.

Exercise 17.1

1. Make r the subject of the formula $C = 2\pi r$.

2. Make c the subject of the formula $a = b - 2c$.

3. Make t the subject of the formula $v = u + at$.

4. In the formula $y = mx + c$,
  i. make c the subject.
  ii. make m the subject.

5. Make c the subject of the formula $a = 2(b + c)$.

6. Make C the subject of the formula $F = \frac{9}{5}C + 32$.

7. In the formula $l = a + (n - 1)d$,
  i. make a the subject.
  ii. make d the subject.

8. Make y the subject of the formula $\frac{x}{a} + \frac{y}{b} = 1$.

9. Make r_2 the subject of the formula $\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2}$.

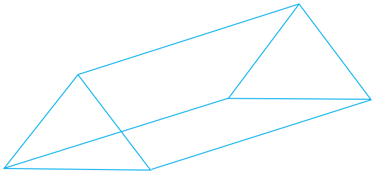
10. Make x the subject of the formula $ax = m(x - t)$.

11. Make a the subject of the formula $P = \frac{at}{a - t}$.

17.2 Substitution

Suppose that the values of all the variables in a formula except one are given. By substituting these values in the formula, the value of the unknown can be found.

Let us determine the number of edges in a solid with straight edges, which has 6 vertices and 5 faces.



The triangular prism shown above is an example of such a solid.

We can find the number of edges by substituting the values $V = 6$ and $F = 5$ in the formula $E = V + F - 2$.

Substituting $V = 6$ and $F = 5$ in the formula we obtain,

$$E = 6 + 5 - 2$$
$$= 9$$

Therefore, the solid has 9 edges.

Let us consider more examples.

There are two methods that can be used to find the value of an unknown variable when the values of the remaining variables are given. The first method is to substitute the given values in the formula as it is, and then find the value of the unknown. The second method is to first make the unknown of which the value is to be determined the subject of the formula, and then find its value by substituting the given values.

Let us now consider how the value of an unknown in a formula is found using these two methods.

Example 1

Determine the number of vertices there are in a solid that has 7 faces and 12 edges.

We need to use the formula $E = V + F - 2$ here. The values of F and E are given and we need to find V. We can use either of the above mentioned two methods to find V. That is, we can first substitute the given values in the formula $E = V + F - 2$ and then find the value of V by solving the resulting equation, or we can first make V the subject of the formula and then substitute the given values and simplify.

Let us consider both methods.
Let us take the number of edges as E, the number of vertices as V and the number of faces as F.

Method 1

$$E = V + F - 2$$

Substituting $E = 12$ and $F = 7$ we obtain

$$12 = V + 7 - 2$$
$$12 = V + 5$$
$$12 - 5 = V$$
$$7 = V$$
$$V = 7$$

$\therefore$ The number of vertices is 7.

Method 2

First make V the subject of the formula and then substitute the values.

$$E = V + F - 2$$
$$E + 2 = V + F$$
$$E + 2 - F = V$$
$$V = E + 2 - F$$
$$V = 12 + 2 - 7$$
$$V = 7$$

$\therefore$ The number of vertices is 7.

Note: One reason for changing the subject of a formula is because the value of the unknown can then be found easily by directly substituting the given values.

Example 2

Convert 35ºC into Fahrenheit using the formula $C = \frac{5}{9}(F - 32)$.
Consider that the temperature in degrees Celsius is denoted by C and the temperature in degrees Fahrenheit is denoted by F.

$$C = \frac{5}{9}(F - 32)$$

Substituting $C = 35$,

$$35 = \frac{5}{9}(F - 32)$$

Multiplying both sides by 9

$$35 \times 9 = 5(F - 32)$$

Dividing both sides by 5

$$\frac{\overset{7}{\cancel{35}} \times 9}{\underset{1}{\cancel{5}}} = F - 32$$

$$63 = F - 32$$

$$63 + 32 = F$$

$$95 = F$$

i.e., $F = 95$

The given tempreature is 95ºF.

Exercise 17.2

1. Find the value of a when $b = 7$ and c = 6 in the formula $a = (b + c) - 2$.
2. Find the value of C when $F = 104$ in the formula $C = \frac{5}{9}(F - 32)$.
3. Find the value of m when $y = 11$, $x = 5$ and $c = -4$ in the formula $y = mx + c$.
4. Find the value of r when $A = 88$ and $\pi = \frac{22}{7}$ in the formula $A = 2\pi r$.
5. Find the value of d when $l = 22$, $a = -5$ and $n = 10$ in the formula $l = a + (n - 1)d$.
6. Find the value of n when $S = -330$, $a = 4$ and $l = -48$ in the formula $S = \frac{n}{2}(a + l)$.

Miscellaneous Exercise

1. Consider the formula $P = C\,(1 + \frac{r}{100})$.

(i) Make r the subject of the above formula.
(ii) Find the value of r when $P = 495$ and $C = 450$.

2. Consider the formula $\frac{y - c}{x} = m$.

(i) Make x the subject of the above formula.
(ii) Find the value of x when $y = 20$, $c = -4$ and $m = 3$.

3. Consider the formula $ax = bx - c$.

(i) Make x the subject of the above formula.
(ii) Find the value of x when $a = 3$, $b = 4$ and $c = 6$.

4. Consider the formula $a = \frac{bx + c}{b}$.

(i) Make b the subject of the above formula.
(ii) Find the value of b when $a = 4$, $c = 5$ and $x = 3$.

5. Find the value of f in the formula $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ when $v = 20$ and $u = 5$.

6. Find the value of b in the formula $\frac{a}{b} = \frac{p}{q}$ when $a = 6$, $p = 3$ and $q = 4$.

7. Consider the formula $S = \frac{n}{2}\,(a + l)$.
(i) Make l the subject of the above formula.
(ii) Find the value of l when $S = 198$, $n = 12$ and $a = 8$.

8. Consider the formula $y = mx + c$.

(i) Make m the subject of the above formula.
(ii) Find the value of m when $y = 8$, $x = 9$ and $c = 2$.

18 Circumference of a Circle

By studying this lesson you will be able to;

- find the diameter of a circle using different methods,
- find the circumference of a circle and the perimeter of a semicircle using formulae,
- solve problems related to the circumference of a circle.

Do the following exercise to recall what you have learnt about circles.

Review Exercise

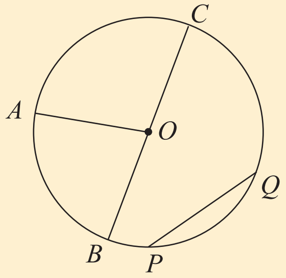
1. a. Fill in the blanks using suitable words.

i. The locus of the points on a plane which are at a constant distance from a fixed point is a. ……………..

ii. The point right at the middle of a circle is known as its ....

b. Copy the two columns A and B given below and using the given figure, join the relevant pairs.

A	B
Point O OA BC OB PQ	Radius Diameter Centre Chord



2.

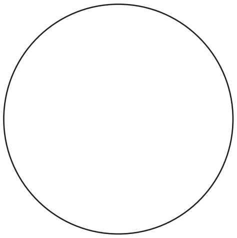
i. What is the length of the diameter of a circle of radius 5 cm?

ii. What is the length of the radius of a circle of diameter 7 cm?

iii. If the radius of a circle is r and diameter is d, write an equation expressing the relationship between d and r.

Measuring the diameter and the circumference of a circle

The total length of the boundary of a circle, or the perimeter of a circle is known as its **circumference**.



A circular ring made from a metal wire of length 25 cm is shown in the above figure. Since the length of the wire is 25 cm, the perimeter or the circumference of the circle is 25 cm.

We cannot directly determine the diameter of a circle.

Do the following activities to identify different methods of finding the diameter of a circle.

Activity 1

(*a*) - Measuring the diameter of a circle using a straight edge with a cm/mm scale.

Step1: Draw any circle using the pair of compasses and mark its centre.

Step2: Draw a diameter and measure its length using a straight edge with a cm/mm scale.

(*b*) - Measuring the diameter by means of an axis of symmetry of a circular lamina.

Step 1: Draw a circle on a piece of paper using an object like a coin or a bangle and cut it out.

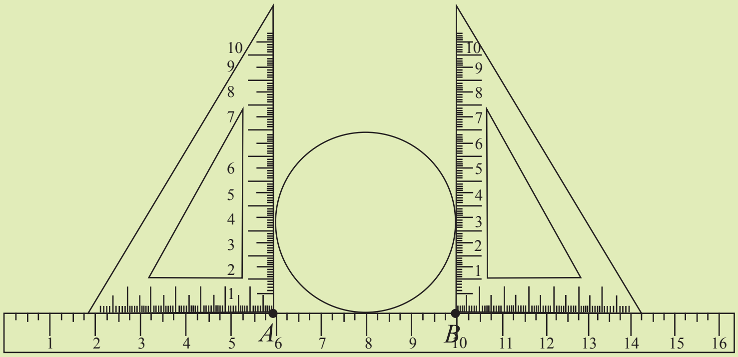
Step 2: Fold the circular lamina into two equal parts (such that the two parts coincide) and mark the axis of symmetry on it.

Step 3: Since the axis of symmetry is a diameter of the circle, measure the length of the axis of symmetry and obtain the length of the diameter.

(*c*)- Measuring the diameter using set squares.

Step 1: Take a ruler, two set squares, a circular coin, a bangle and a cylindrical can.

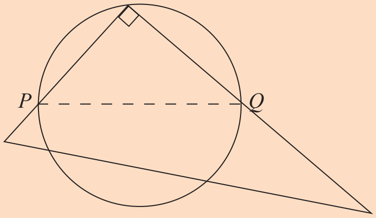
Step 2: Place the bangle and the set squares as shown in the figure, touching the ruler. Find the diameter of the bangle using the two readings denoted by *A* and *B*.



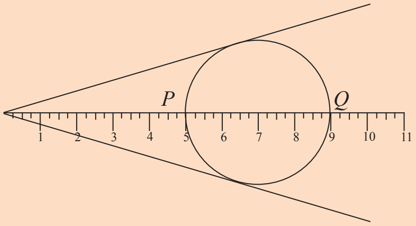
Step 3: Find the diameters of the remaining circular objects too by doing the above activity and note them down in your exercise book.

Different methods of finding the diameter

1. Keep the right angled corner of a piece of paper on a circle as shown in the figure. The distance between the two points(*P* and *Q*) where the arms of the 90° angle meet the circle is the length of the diameter of the circle.



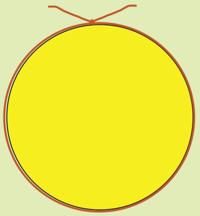
2. Make an instrument as shown in the figure: Draw an angle and its bisector on a Bristol board and calibrate the bisector from the vertex. The length of the diameter of a circle can be measured as shown in the figure.



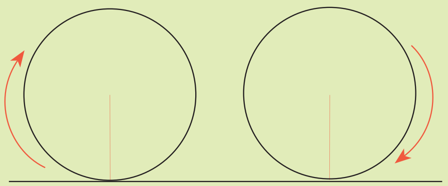
18.1 Measuring the circumference of a circle

Do the following activities in order to find out the methods used to measure the circumference of a circular lamina such as a coin.

Activity 2



1. Mark a point on a piece of thread and with the thread stretched, place it around the circular lamina until it reaches the initial point that was marked. Mark the final point on the thread when it coincides with the initial point, and measure the length between the two points marked on the thread to find the circumference of the coin.



2. Draw a straight line on a piece of paper. Keep it on a flat surface. Mark a point on the circumference of the circular lamina and also on the straight line. Place the circular lamina on the straight line such that the two marks coincide and then role it along the straight line until the point marked on the circular lamina touches the line again. The length of the circumference is obtained by measuring the distance the circular lamina has moved along the straight line.

Developing a formula for the circumference of a circle

Do the following activity to identify the relationship between the circumference and the diameter of a circle.

Activity 3

Complete the table given below by measuring the circumference and the diameter of objects with circular faces, using the methods introduced above.

Object	Diameter d	Circumference c	$\frac{c}{d}$ up to two decimal places
1. Circular lamina made of cardboard			
2. Rs 2 coin			
3. Circular lid of a tin			
4. Compact Disk (CD)			

Compare the values you obtained for $\frac{c}{d}$ in the above activity with the values obtained by your friends and write your conclusion regarding the value of $\frac{c}{d}$.

Through the above activity you would have obtained a value for $\frac{c}{d}$ which is approximately 3.14 for every object you considered. Mathematicians have discovered that $\frac{c}{d}$ is a constant value for all circles. This constant value is denoted by π. It has been shown that this value is 3.14 to the nearest second decimal and is approximately equal to the fraction $\frac{22}{7}$.

Accordingly,

$$\frac{c}{d} = \pi \,.$$

That is,

$$c = \pi d.$$

This is a formula giving the relationship between the circumference and the diameter of a circle. A formula giving the relationship between the radius and the circumference of a circle can be derived as follows.

Since $d = 2r$ we obtain $c = \pi \times 2r.$

i,e.,

$$c = 2\pi r$$

If the circumference of a circle is denoted by c, the diameter by d and the radius by r, then,

$$\boxed{c = \pi d}$$

$$\boxed{c = 2\pi r}$$

Example 1

Find the circumference of a circle of radius 7 cm.

Use $\frac{22}{7}$ for the value of π.

Circumference $c = 2\pi r$

$$= 2 \times \frac{22}{\cancel{7}} \times \overset{1}{\cancel{7}}$$

$$= 44$$

$\therefore$ the circumference is 44 cm.

Exercise 18.1

1. Find the circumference of the circle with the measurement given below.

Use $\frac{22}{7}$ for the value of π.

i. radius 7 cm

ii. diameter 21 m

iii. radius 10.5 cm

iv. diameter $17\frac{1}{2}$ m

v. radius $\frac{7}{2}$ m

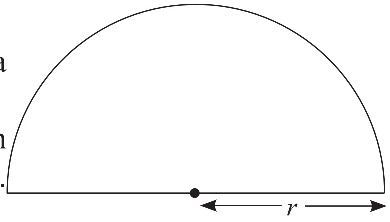
vi. diameter 28 cm

vii. radius 15.4 cm

viii. diameter $3\frac{1}{9}$ m

18.2 Perimeter of a semicircular lamina

When a circular lamina is separated into two equal parts along a diameter, each part is known as a semicircular lamina (in short, semicircle).
The length of the curved line of a semicircle is known as the arc length. It is exactly half the circumference.



Hence, the arc length of a semicircle of radius $r = \frac{1}{\cancel{2}} \times (\overset{1}{\cancel{2}}\pi r)$

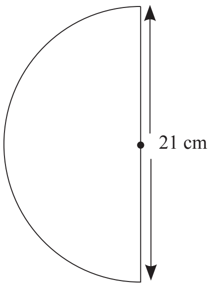
$= \pi r$

It is clear from the figure that, to find the perimeter of a semicircle, the diameter should be added to the arc length.

$$\boxed{\therefore \text{ The perimeter of a semicircle} = \pi r + 2r}$$

Example 1

Find the perimeter of the semicircle shown in the figure. Use $\frac{22}{7}$ for the value of π.



Arc length of a semicircle of diameter $d = \frac{1}{2}\pi d$

$\therefore$ Arc length of the semicircle of diameter 21 cm $= \frac{1}{\underset{1}{\cancel{2}}} \times \frac{\overset{11}{\cancel{22}}}{\underset{1}{\cancel{7}}} \times \overset{3}{\cancel{21}}$

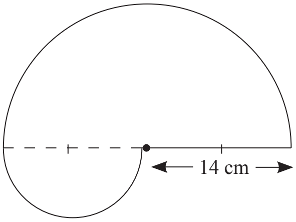
$= 33$

$\therefore$ The perimeter of the figure $= 33 + 21 = 54$ cm

Example 2

A compound figure, consisting of two semicircular laminas of radius 14 cm and diameter 14 cm respectively is shown in the figure. Find its perimeter.

Use $\frac{22}{7}$ for the value of π.



$$\text{Arc length of a semicircle of radius } r = \frac{1}{2} \times 2\pi r$$

$$\therefore \text{ Arc length of the semicircle of radius 14 cm} = \frac{1}{\underset{1}{\cancel{2}}} \times \overset{1}{\cancel{2}} \times \frac{22}{\underset{1}{\cancel{7}}} \times \overset{2}{\cancel{14}} \text{ cm} = 44 \text{ cm}$$

$$\text{Arc length of a semicircle of diameter } d = \frac{1}{2}\pi d$$

$$\therefore \text{ Arc length of the semicircle of diameter 14 cm} = \frac{1}{\underset{1}{\cancel{2}}} \times \frac{\overset{11}{\cancel{22}}}{\underset{1}{\cancel{7}}} \times \overset{2}{\cancel{14}} \text{ cm} = 22 \text{ cm}$$

$$\therefore \text{Perimeter of the figure} = 44 + 22 + 14 \text{ cm}$$

$$= \underline{\underline{80 \text{ cm}}}$$

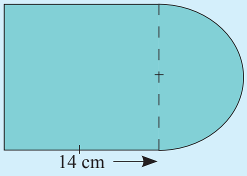
Exercise 18.2

1. Find the perimeter of the semi circular lamina with the measurement given below. Use $\frac{22}{7}$ for the value of π.

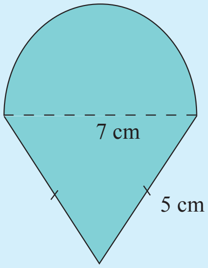
i. r =14cm   ii. d =7 cm

2. Find the permeter of the shaded part of each of the figures given below. The curved parts in the figures are semicireles.Use $\frac{22}{7}$ for the value of π.

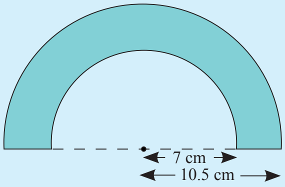
i.



ii.



iii.



18.3 Problems related to the circumference of a circle

Example 1

A wheel of radius 35 cm moves along a straight road.

i. Find in metres, the distance it moves during one full rotation.

ii. What is the distance it moves in metres during 100 rotations?

iii. How many rotation does the wheel undergo when travelling a distance of 1.1 km? (Use $\frac{22}{7}$ for the value of π.)

i. During one full rotation of the wheel, it moves a distance which is equal to its circumference.

$$\text{Circumference} = 2 \times \frac{22}{\underset{1}{\cancel{7}}} \times \overset{5}{\cancel{35}}\ \text{cm} = 220\ \text{cm}$$

$\therefore$ The distance travelled during one full rotation = $\underline{\underline{2.2\ \text{m}}}$

ii. Distance travelled during 100 rotations $= 2.2\ \text{m} \times 100$

$= \underline{\underline{220\ \text{m}}}$

iii. Distance travelled = 1.1 km

= 1100 m

Distance travelled during one rotation of the wheel = 2.2 m

Therefore, number of rotations $= \frac{1100}{2.2}$

$= \underline{\underline{500}}$

Example 2

A circular frame is made by joining the two ends of a wire of length 66 cm. Find its radius.

Let us assume that the radius is r cm.

$$c = 2\pi r$$

$$2 \times \frac{22}{7} \times r = 66$$

$$r = \overset{3}{\cancel{66}} \times \frac{7}{\underset{1}{\cancel{22}}} \times \frac{1}{2}$$

$$= \frac{21}{2}$$

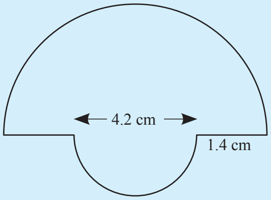
$$= 10.5 \text{ cm}$$

∴ The radius is 10.5 cm.

Exercise 18.3

Use $\frac{22}{7}$ for the value of π.

1. A lamina composed of two semicircles is shown in the figure. This is pasted on an ornamental box. A gold thread is to be pasted around the lamina.



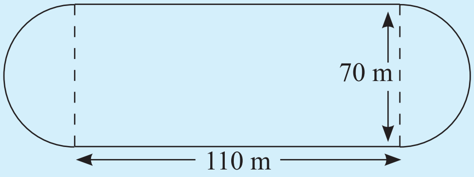
(i) Find the minimum length of thread required to paste around this lamina?

(ii) Find the total length in metres of the thread required to paste around 500 such laminas.

2. The circumference of a circular plot of land is 440 m. Find its radius.

3. The perimeter of a semicircular lamina is 39.6 cm. Find its diameter.

4. A sketch of a playground is shown in the figure. It consists of a rectangular part and two semicircular parts.



(i) Find the perimeter of the playground.

(ii) Show that the distance covered by a runner in completing $2\frac{1}{2}$ rounds of the playground is more than 1 km.

5. A cyclist rides a bicycle along a straight road. The radius of each wheel of the bicycle is 28 cm.

(i) Find the distance the bicycle moves during the period athat the wheels complete one full rotation.

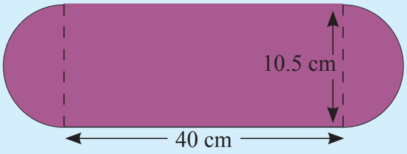
(ii) What is the distance the bicycle moves in meters during the period that the wheels complete 50 rotation?

(iii) The cyclist says that the wheels rotate at least 800 times when it travels a distance of 1500 m. Do you agree with this statement? Give reasons for your answer.

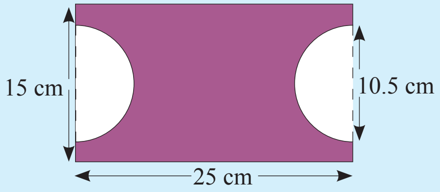
Miscellaneous Exercise

1. Find the perimeter of the shaded part of each figure.

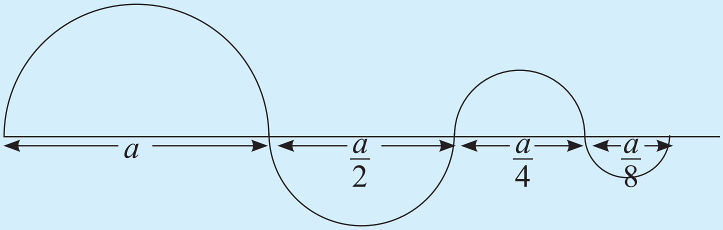
i.



ii.

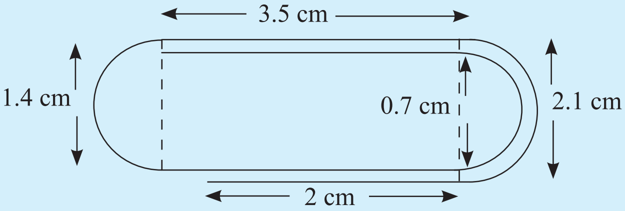


2.



Show that the length of metal wire needed to make the frame shown in the figure which consists of 4 semicircular parts is $\frac{135a}{28}$. (Use $\frac{22}{7}$ for the value of π.)

3. A paper clip with semi circular parts is to be made according to the given measurements. Find the length of the wire needed to make the clip in the figure.



Summary

In acircle of radius r, diameter d and circumference c,

- $c = \pi d$
- $c = 2\pi r$
- The perimeter of a semicircle = $\pi r + 2r$

19 Pythagorean Relation

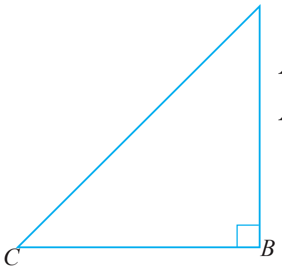
By studying this lesson, you will be able to;

- develop the Pythagorean relation by means of a right angled triangle,
- solve problems related to the Pythagorean relation.

Right angled triangle

If an angle of a triangle is 90°, it is called a right angled triangle. The side which is opposite(in front of) the right angle and which is the longest side of the triangle is called the hypotenuse. The other two sides are called the sides which include the right angle.

Considering the right angled triangle ABC given below;



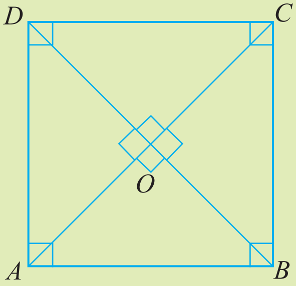
$A\hat{B}C = 90^{\circ}$

AC is the hypotenuse,

AB and BC are the sides which include the right angle.

Activity 1

Complete the table given below by identifying all the right angled triangles in the figure.

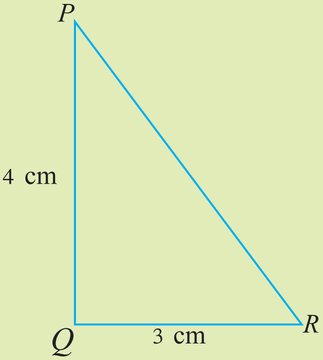


Triangle	Hypotenuse	Sides that include the right angle
AOB	AB	AO, BO
....	....	....
....	....	....
....	....	....
....	....	....
....	....	....

19.2 The Pythagorean relation

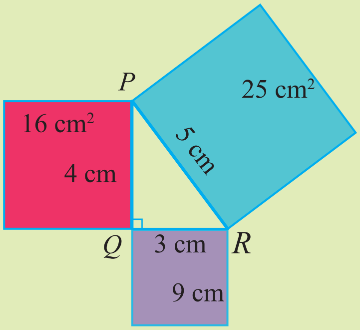
A Greek mathematician named Pythagoras introduced the relationship between the sides of a right angled triangle. Let us understand this relationship through an activity.

Activity 1



Draw a right angled triangle PQR with $QR = 3$ cm and $QP = 4$ cm. You may use a set square to do this. Measure the hypotenuse PR and verify that it is 5 cm in length. Cut three squares of side length 3 cm, 4 cm and 5 cm and paste them on the sides RQ, QP and PR respectively, as shown in the figure given below.

Now let us calculate the area of each square as shown below.



The area of the square pasted on $QR = 3\text{ cm} \times 3\text{ cm} = 9\text{ cm}^2$
The area of the square pasted on $QP = 4\text{ cm} \times 4\text{ cm} = 16\text{ cm}^2$
The area of the square pasted on $PR = 5\text{ cm} \times 5\text{cm} = 25\text{ cm}^2$

Observe the relationship between the above values as given below.

the area of the square on PR = the area of the square on QR + the area of the square on PQ

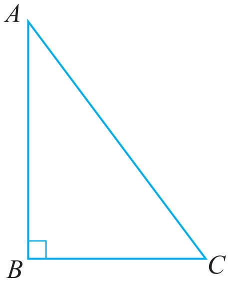
Repeat the above activity by taking the lengths of the sides of the triangle which include the right angle as 6 cm and 8 cm to verify the above relationship for these values too.

The Pythagorean relation for a right angled triangle can be expressed as follows.

The area of the square drawn on the hypotenuse of a right angled triangle is equal to the sum of the areas of the squares drawn on the remaining two sides.

Though the Pythagorean relation is shown using areas, we can write it simply in terms of the lengths of the sides of the triangle. Let us see how this is done.

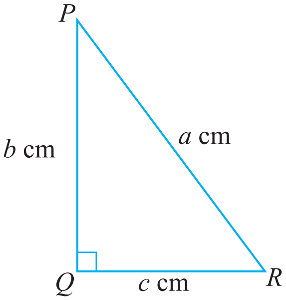
Writing the Pythagorean relation in terms of the lengths of the side



The area of the square drawn on $AB = AB \times AB = AB^2$
The area of the square drawn on $BC = BC \times BC = BC^2$
The area of the square drawn on $AC = AC \times AC = AC^2$
Therefore, according to the Pythagorean relation;

$$AC^2 = AB^2 + BC^2$$

We can express this as follows too.



According to the Pythagorean relation
$a^2 = b^2 + c^2$

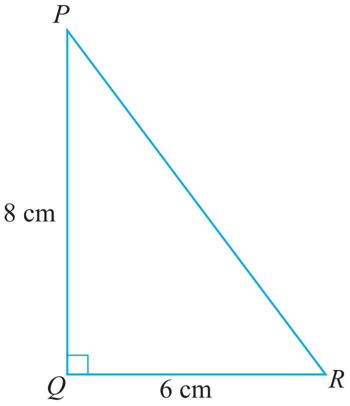
Example 1

In the right angled triangle PQR, $PQ = 8$ cm and $QR = 6$ cm. Find the length of PR.

By applying the Pythagorean relation to the right angled triangle PQR,

$$PR^2 = PQ^2 + QR^2$$
$$PR^2 = 8^2 + 6^2$$
$$= 64 + 36$$
$$= 100$$
$$PR = \sqrt{100} = 10$$

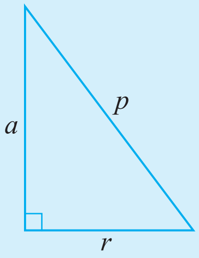
$\therefore$ the length of $PR = 10$ cm.



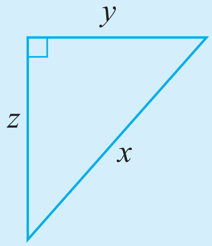
Exercise 19.1

1. Write the Pythagorean relation for each right angled triangle using the given lengths.

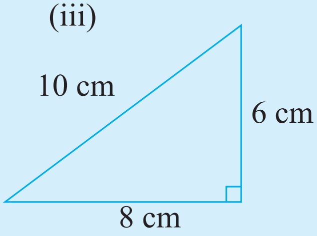
(i)



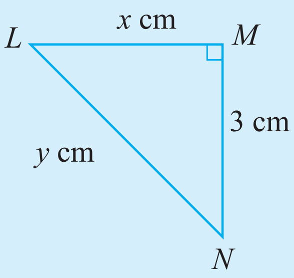
(ii)



(iii)

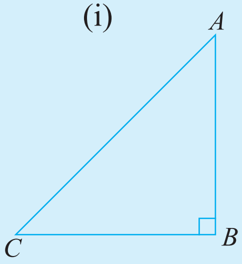


(iv)



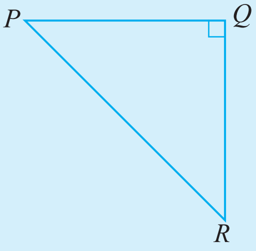
2. Fill in the blanks in the expressions related to the figures shown below.

(i)



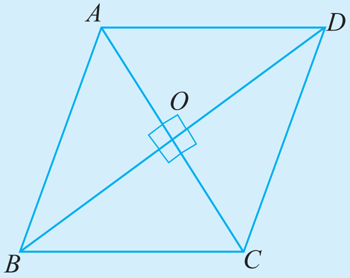
$AC^2 = AB^2 +$ ....

(ii)



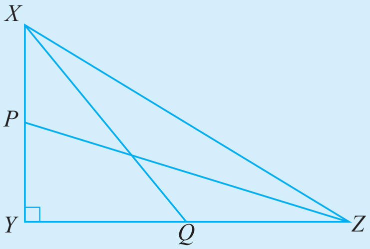
$PR^2 =$ .... + ...

(iii)

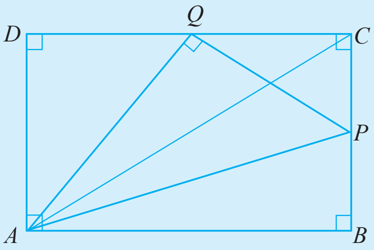


a. $AD^2 =$ .... + ...
b. .... $= BO^2 +$ ....
c. .... $= BO^2 + OC^2$
d. $DC^2 =$ .... + ...

3. Identify all the right angled triangles in each figure given below and write the Pythagorean relation for each triangle that is identified.

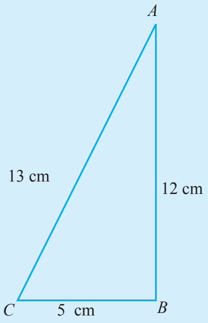


i.



ii.

4. Fill in the blanks in the statements given below which are related to the right angled triangle in the figure.



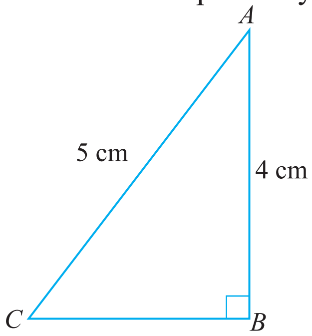
The longest side of the triangle is ....
The area of the square drawn on the side AB = $12 \times 12 = 144$ cm^2
The area of the square drawn on the side BC = ... = ... cm^2
The area of the square drawn on the side AC = ... = ... cm^2
The sum of the areas of the squares drawn on the sides BC and BA = ... cm^2 .
∴ The areas of the square drawn on AC is .... (equal/not equal) to the sum of the areas of the squares drawn on BC and BA.

Now let us consider some problems which can be solved using the Pythagorean relation.

Example 2

A 5m long straight wooden rod is in a vertical plane with one end touching the top of a 4m high vertical wall and the other end in contact with the horizontal ground a certain distance away from the foot of the wall. Find the horizontal distance from the foot of the wall to the point where the rod is in contact with the ground.

We can draw a rough sketch of this as shown below. Here the wall and the wooden rod are represented by BA and AC respectively.



Applying the Pythagorean relation to the right angled triangle ABC

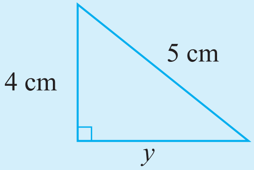
$$AC^2 = AB^2 + BC^2$$
$$5^2 = 4^2 + BC^2$$
$$25 = 16 + BC^2$$
$$\therefore BC^2 = 9$$
$$BC = \sqrt{9} = 3$$

∴ The horizontal distance from the foot of the wall to the wooden rod is 3m.

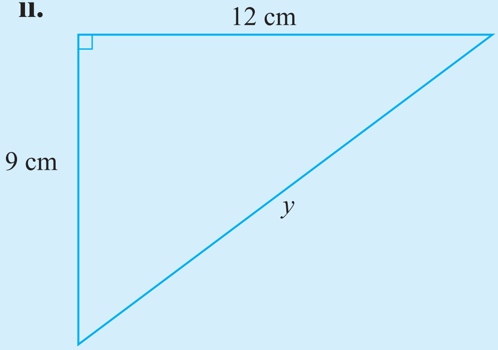
Exercise 19.2

1. Find the length of each side indicated by an algebraic symbol in each figure.

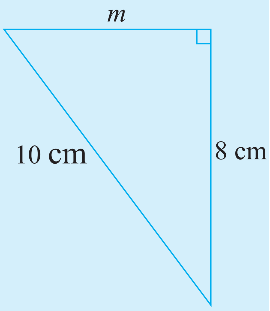
i.



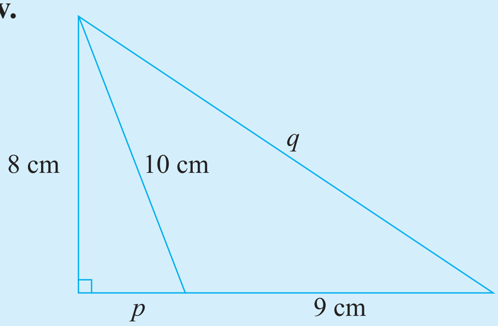
ii.



iii.

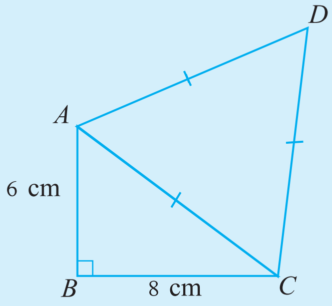


iv.

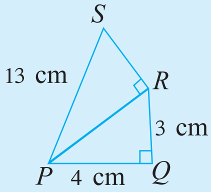


2. Find the perimeter of each figure given below.

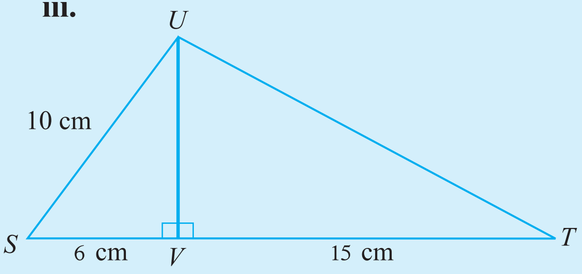
i.



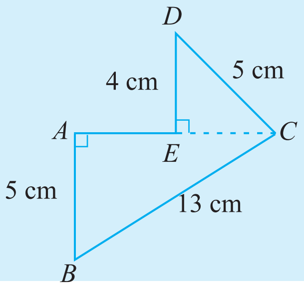
ii.



iii.

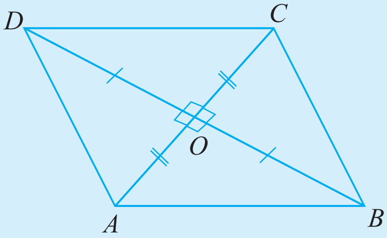


iv.



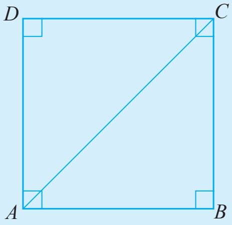
3.

i.



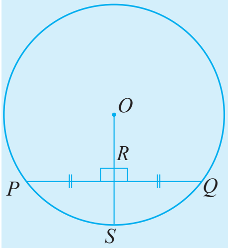
The diagonals *BD* and *AC* of the rhombus *ABCD* bisect each other perpendicularly at *O*. Moreover, $BD = 16$cm and $AC = 12$cm. Find the perimeter of the rhombus.

ii.



If the length of the diagonal *AC* of the square *ABCD* is 10cm, find the area of the square.

iii.



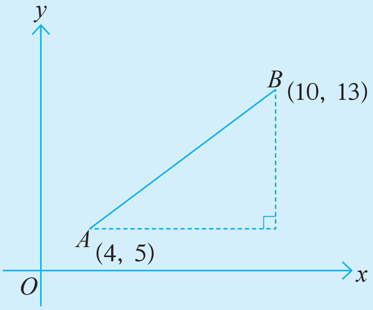
In the circle with centre *O* shown in the figure, the midpoint of the chord *PQ* is *R*. Moreover, *OR* produced meets the circle at *S*. If $O\hat{R}P = 90^{\circ}, O\hat{R}P$ PQ=12 cm and OR= 8 cm, find

i. the length of *RQ,*
ii. the radius of the circle,
iii. the length of *RS.*

4. In the triangle *ABC*, $A\hat{B}C = 90^{\circ}$, $AB = 8$ cm and $BC = 6$ cm. The mid points of *BC* and *BA* are *R* and *P*. Find the perimeter of the quadrilateral *APRC*.

Miscellaneous Exercise

1.

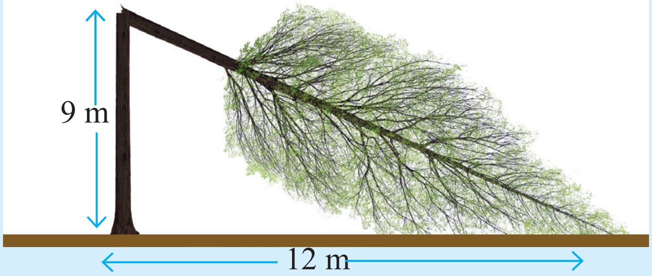


Find the shortest distance between the point $A = (4, 5)$ and $B = (10, 13)$ located on a Cartesian plane.

2. The city Q is located 5 km east of the city P and the city R is located 12 km north of the city Q. Find the distance between the two cities P and R.

3. To keep a 16m tall flag post vertical, one end of a supportive cable is attached to the top of the post while the other end is fixed to a point on the ground (horizontal), 12m from the foot of the flag post. Another cable is fixed from the opposite direction, with one endattached to the flag post, 12m above its foot, and the other to the ground, 9m from its foot. Calculate the total length of the cable that has been used.

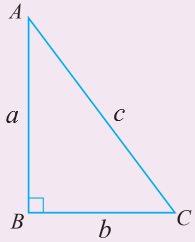
4.



A tree which was struck by a tornado is shown in the figure. Find the height of the tree before it was struck.

Summary

In the ABC right angled triangle



$$AC^2 = AB^2 + BC^2$$

$$c^2 = a^2 + b^2$$

20

Graphs

By studying this lesson you will be able to;

- identify functions,
- draw graphs of functions of the form $y = mx$ and $y = mx + c$ and identify their characteristics,
- identify the gradient and intercept of a straight line graph,
- plot straightline graphs of equations of the form $ax + by = c$
- identify the relationship between the gradients of straight lines which are parallel to each other.

Do the following exercise to recall what has been learnt in previous grades regarding graphs.

Review Exercise

1. i. Draw a coordinate plane with the x and y axes marked from – 5 to 5 and mark the points A $(-4, -4)$ and B $(4, -4)$ on it. Mark the points C and D such that $ABCD$ is a square and write the coordinates of C and D.

ii. Write the equation of each side of the plane figure $ABCD$.

2. Draw a coordinate plane with the x and y axes marked from – 4 to 4.

i. Draw two straight lines, one parallel to the x axis and the other parallel to the y axis passing through the point $(4, -4)$.

ii. Draw another two straight line, one parallel to the x axis and the other parallel to the y- axis passing through the point $(-3, 2)$.

iii. Write the coordinates of the two points at which the lines in (i) and (ii) above intersect each other.

iv. Write the equations of the axes of symmetry of the plane figure obtained in (iii) above.

20.1 Functions

We have come across relationships between different quantities in various situations. Carefully observe the relationship between the two quantities given below.

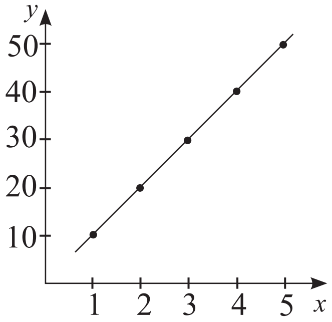
Let us suppose that beads are sold at 10 rupees per gramme. The prices of different amounts of beads is shown below.

Beeds (g)		Price (Rs)
1	$\longrightarrow$	$1 \times 10 = 10$
2	$\longrightarrow$	$2 \times 10 = 20$
3	$\longrightarrow$	$3 \times 10 = 30$
4	$\longrightarrow$	$4 \times 10 = 40$

Accordingly, it is clear that the price of x grammes of beeds is Rs $10x$. Moreover, if the price of x grammes of beeds is represented by Rs y, then we can write $y = 10x$.

Let us take the amount of grammes of beeds as x and the corresponding price as Rs y.

By plotting the price (y) against the amount of grammes of beeds (x) for different values of x using the above relationship, we obtain the following graph.



In the above function $y = 10x$, the index of the independent variable x is 1. Therefore it is called a linear function.

When a linear function is given, the values of y corresponding to different values of x can be found as follows.

Example 1

For each of the linear functions given below, calculate the values of y corresponding to the given values of x and write them as ordered pairs.

i. $y = 2x$ (values of x: $-2, -1, 0, 1, 2$)
ii. $y = -\frac{3}{2}x + 2$ (values of x: $-4, -2, 0, 2, 4$)

i. $y = 2x$

x	$2x$	y	Ordered pairs (x, y)
-2	2×-2	-4	$(-2, -4)$
-1	2×-1	-2	$(-1, -2)$
0	2×0	0	$(0, 0)$
1	2×1	2	$(1, 2)$
2	2×2	-4	$(2, 4)$

ii. $y = -\frac{3}{2}x + 2$

x	$-\frac{3}{2}x + 2$	y	Ordered pairs (x, y)
-4	$-\frac{3}{2} \times -4 + 2$	8	$(-4, 8)$
-2	$-\frac{3}{2} \times -2 + 2$	5	$(-2, 5)$
0	$-\frac{3}{2} \times 0 + 2$	2	$(0, 2)$
2	$-\frac{3}{2} \times 2 + 2$	-1	$(2, -1)$
4	$-\frac{3}{2} \times 4 + 2$	-4	$(4, -4)$

Exercise 20.1

Find the values of y corresponding to the given values of x and write them as ordered pairs.

i. $y = 3x$ (values of x: $-2, -1, 0, 1, 2$)

ii. $y = 2x + 3$ (values of x: $-3, -2, -1, 0, 1, 2, 3$)

iii. $y = -\frac{1}{3}x - 2$ (values of x: $-6, -3, 0, 3, 6$)

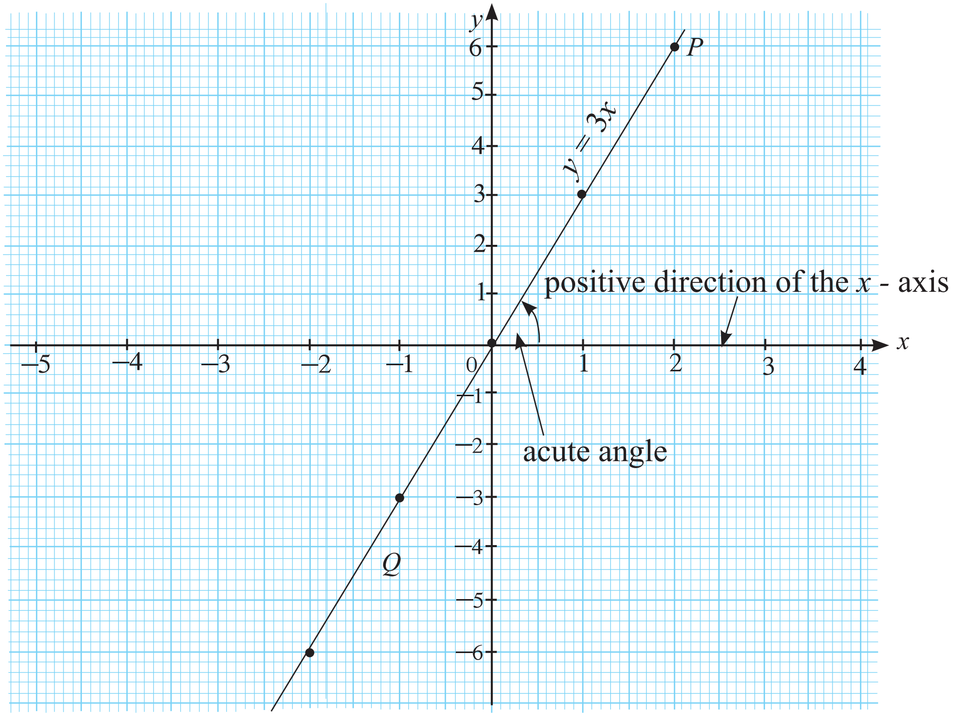
20.2 Functions of the form $y = mx$ and the gradient of the graph of such a function

Linear functions such as $y = 3x$, $y = -2x$ and $y = x$ are examples of functions of the form $y = mx$. Let us obtain ordered pairs of values of x and y by constructiong a table as follows, to draw the graph of $y = 3x$ for the values of x from -2 to $+2$.

$y = 3x$

x	$3x$	y	(x, y)
-2	3×-2	-6	$(-2, -6)$
-1	3×-1	-3	$(-1, -3)$
0	3×0	0	$(0, 0)$
1	3×1	3	$(1, 3)$
2	3×2	6	$(2, 6)$

By plotting the above ordered pairs in a coordinate plane, we obtain the graph of the function $y = 3x$ as shown below.



Let us consider some characteristics of the above drawn graph.

- The graph is a straight line
- It passes through the point (0, 0)
- It makes an acute angle counterclockwise with the positive direction of the x - axis
- When any point on the line other than the origin is considered, the value of $\frac{y \text{ coordinate}}{x \text{ coordinate}}$ of that point is a constant (a constant value).

For example,

when the point P is considered, $\frac{y \text{ coordinate}}{x \text{ coordinate}} = \frac{6}{2} = 3$

when the point Q is considered, $\frac{y \text{ coordinate}}{x \text{ coordinate}} = \frac{-3}{-1} = 3$

Moreover, this constant value is equal to the coefficient m of x of a function of the form $y = mx$. This constant value is called the **gradient** of the graph.

The gradient can be a positive value or a negative value.

Now, let us explain the behavior of functions of the form $y = mx$ through the activity given below.

Activity 1

1. a. Complete the tables given below for the selected positive values of m to obtain coordinates of points to draw the graphs of the given functions of the form $y = mx$, and draw the graphs on one coordinate plane.

(i) $y = x$ (ii) $y = +3x$ (iii) $y = +\frac{1}{3}x$

x	-2	0	2
y	___	___	$+2$

x	-1	0	1
y	-3	___	___

x	-3	0	3
y	___	___	$+1$

b. Complete the tables given below for the selected negative values of m to obtain coordinates of points to draw the graphs of the given functions of the form $y = mx$ and draw the graphs on one coordinate plane.

(i) $y = -x$ (ii) $y = -3x$ (iii) $y = -\frac{1}{3}x$

x	-2	0	2
y	___	___	-2

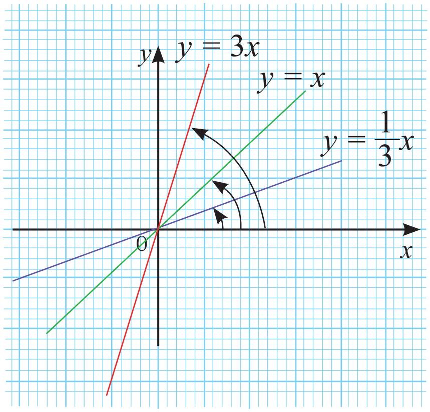
x	-1	0	1
y	___	0	___

x	-3	0	3
y	1	___	___

Observe the relationship between the angle that a graph makes with the positive direction of the x -axis and the gradient (value of m) by considering the graphs obtained in (a) and (b) above.

By doing the above activity you would have obtained the following graphs.

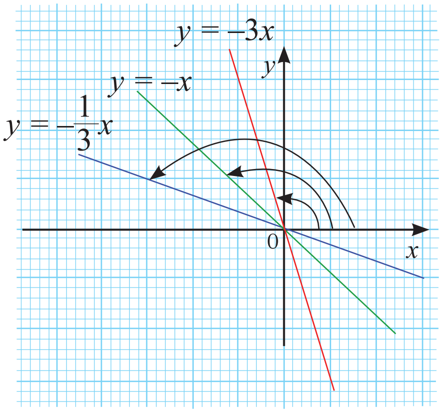
(a) Graphs obtained when the gradient is positive



★ When the gradient (value of m) is positive, the angle that the graph forms counter clockwise with the positive direction of the x-axis is an acute angle.

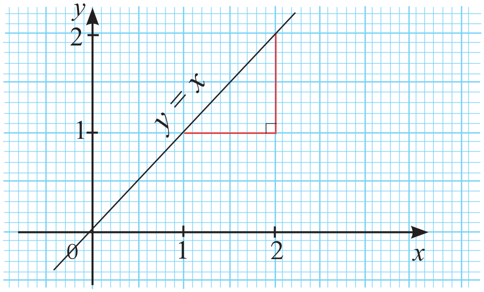
- ★ As the value of the gradient increases in the order $\frac{1}{3}$, 1, 3 the magnitude of the angle formed by the graph of the function with the positive direction of the x-axis counterclockwise increases.

(b) Graphs obtained when m is negative

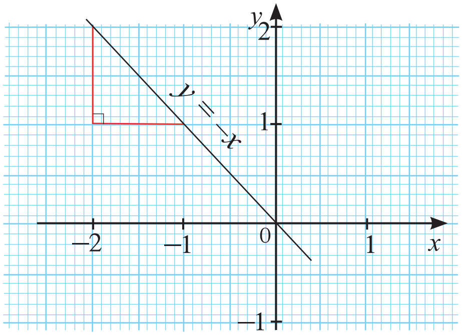


- ★ When the value of the gradient (value of m) is negative, the angle formed with the positive direction of the x-axis counterclockwise is an obtuse angle.
- ★ As the value of the gradient (value of m) increases in the order $-3, -1, -\frac{1}{3}$ the magnitude of the angle formed by the graph of the function with the positive direction of the x-axis counterclockwise increases.

Note: Gradient of a graph



The gradient of the graph of the function $y = x$ is 1. This means that when the value of x increases by one unit, the value of y also increases by one unit.



The gradient of the graph of the function $y = -x$ is –1. This means that when the value of x increases by one unit, the value of y decreases by one unit.

Example 1

Write the gradient of the graph of each of the functions given below without drawing the graph.

i. $y = 2x$
ii. $y = -5x$
iii. $y = -\frac{1}{2}x$

i. Gradient $(m) = 2$
ii. Gradient $(m) = -5$
iii. Gradient $(m) = -\frac{1}{2}$

Example 2

i. Draw the graphs of the functions $y = 2x$ and $y = -3x$ on the same coordinate plane by selecting suitable values for x.

ii. Using the above drawn graphs, find the value of x when $y = 3$, and the value of y when $x = 2$.

i. $y = 2x$

x	-2	-1	0	1	2
$+2x$	2×-2	2×-1	2×0	2×1	2×2
y	-4	-2	0	2	4

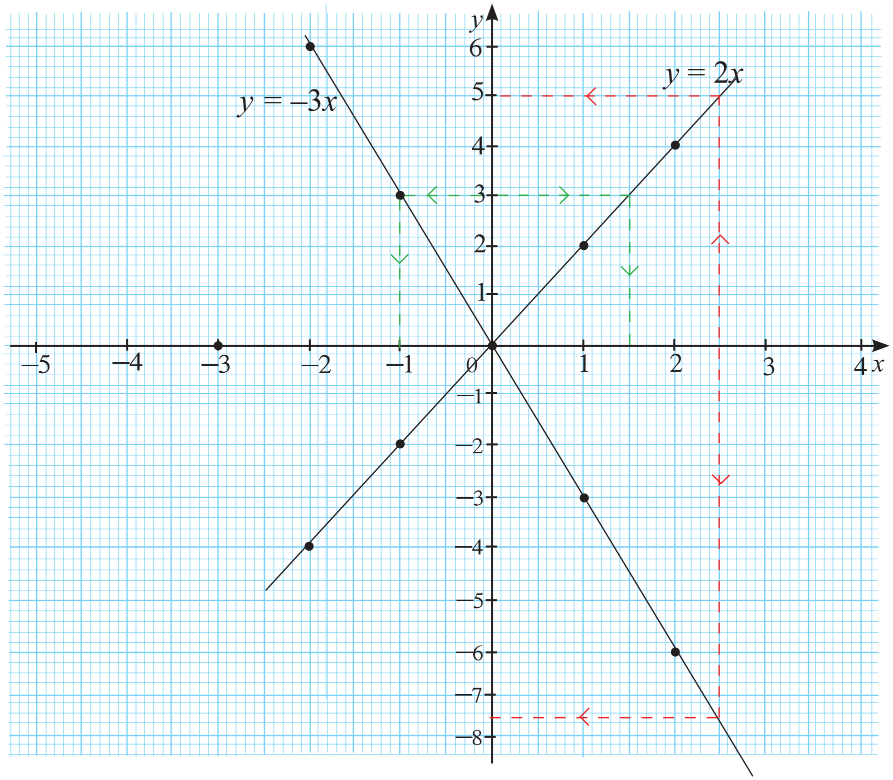
$(-2, -4)$ $(-1, -2)$ $(0, 0)$ $(1, 2)$ $(2, 4)$

$y = -3x$

x	-2	-1	0	1	2
$-3x$	-3×-2	-3×-1	-3×0	-3×1	-3×2
y	6	3	0	-3	-6

$(-2, 6)$ $(-1, 3)$ $(0, 0)$ $(1, -3)$ $(2, -6)$

By plotting the above ordered pairs on the same coordinate plane, graphs of the following form will be obtained.



ii. To find the value of y when $x = 2.5$, the line $x = 2.5$, should be drawn (indicated by the red line) and the y -coordinate of the points of intersection of this line with the two graphs should be obtained.

If we consider the function $y = 2x$, the value of y is 5 when the value of x is 2.5

If we consider the function $y = -3x$, the value of y is -7.5 when the value of x is 2.5

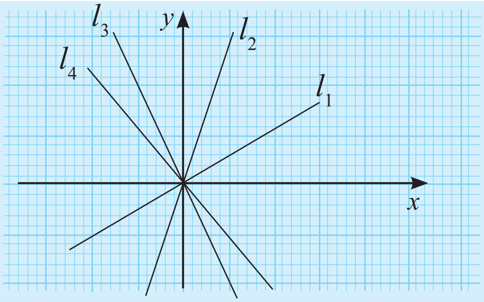
To find the value of x when $y = 3$, the line y = 3 should be drawn (indicated by the green line) and the x - coordinate of the points of intersection of this line with the two graphs should be obtained.

If we consider the function $y = 2x$, the value of x is $1\frac{1}{2}$ when the value of y is 3.

If we consider the function y = $-3x$, the value of x is -1 when the value of y is 3.

Exercise 20.2

1. For each of the functions given by the following equations, select and write the corresponding graph, from the graphs l_1, l_2, l_3, and l_4 .



i. $y = 3x$ **ii.** $y + 2x = 0$

iii. $2y - x = 0$ **iv.** $y + \frac{3}{2}x = 0$

2. The value of a Singapore dollor in Sri Lankan rupees is 100. By taking the amount of Singapore dollors as x and the corresponding value in Sri Lankan rupees as y, the relationship between Singapore dollors and Sri Lankan rupees can be written as a function as $y = 100x$.

i. Prepare a suitable table of values to draw the graph of the above function. (Use the values 1, 2, 3 and 4 for x)

ii. Draw the graph of the above function.

iii. Using the above drawn graph, find the value of 4.5 Singapore dollors in Sri Lankan rupees.

iv. Using the graph, find how much 250 Sri Lankan rupees are in Singapore dollors.

3. For the statements given below, mark a '✓' in front of the correct statements and a '✗' in front of the incorrect statements.

i. For functions of the form $y = mx$, the direction of the graph is decided by the sign of m. (....)

ii. When the graph of a function of the form $y = mx$ is given, the graph of $y = -mx$ cannot be obtained by considering symmetry about the y - axis. (....)

iii. For a straight line passing through the origin, the ratio of the y - coordinate to the x - coordinate of any point on the line other than the origin is equal to its gradient. (....)

iv. Although the point $(-2, 3)$ lies on the straight line given by $2y + 3x = 0$, it does not lie on the straight line given by $2y - 3x = 0$. (....)

v. The graph of a function of the form $y = mx$ need not pass through the point (0,0). (....)

4. i. Construct a table of values to draw the graphs of the functions given by $y = \frac{1}{3}x$, $3y = 2x$ and $y = -1\frac{1}{3}x$ by taking the values of x to be – 6,– 3, 0, 3 and 6.

ii. Draw the above graphs on the same coordinate plane.

iii. Write the x - coordinate of each of the three points at which the line $y = 1$ intersects the above three graphs.

5. i. Fill in the blanks in the incomplete table given below to draw the graph of the function $y = -\frac{2}{3}x$.

x	– 6	– 3	0	3	6
y	4	____	____	– 2	____

ii. Draw the graph of the above given function using the values in the completed table.

iii. Using the graph, find the value of y when $x = -2$.

iv. Does the point $(-\frac{2}{3}, \frac{2}{3})$ lie on the above graph? Explain your answer with reasons.

v. Using the coordinates of three points on the line, calculate the ratio of the y - coordinate to the x - coordinate. Write the relationship between this value and the gradient.

20.3 Graphs of functions of the form $y = mx + c$ and functions given by $ax + by = c$

Graphs of functions of the form $y = mx + c$

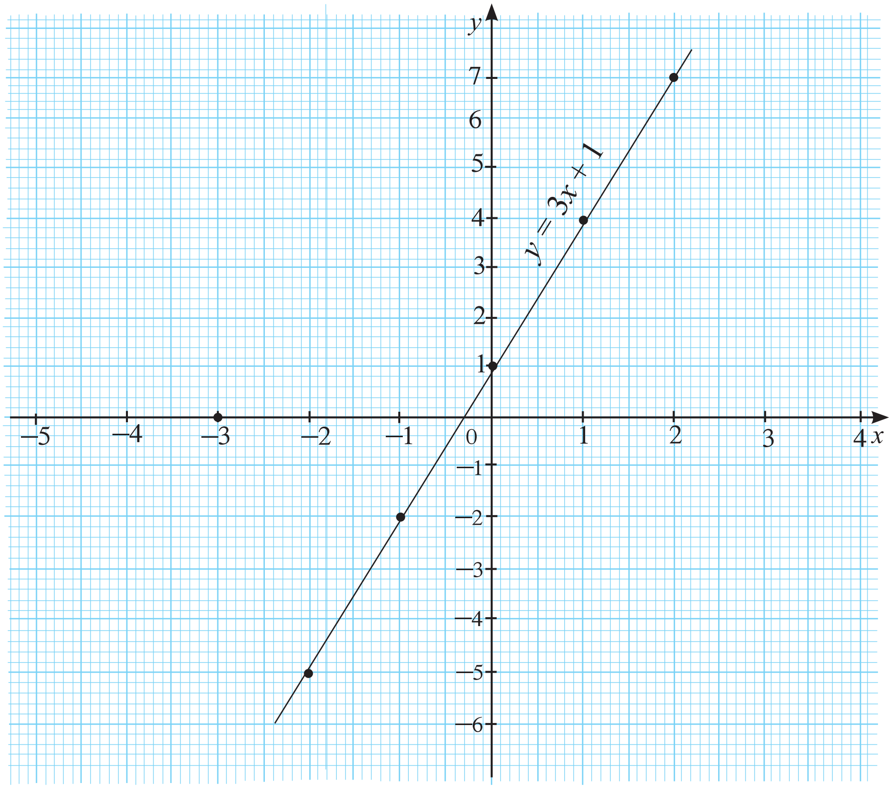
Let us first consider functions of the form $y = mx + c$. Let us draw the graph of the function $y = 3x + 1$.

To do this, let us first develop a lable of values as shown below.

$y = 3x + 1$

x	$3x + 1$	y	(x, y)
– 2	$3 \times -2 + 1$	– 5	(– 2, – 5)
– 1	$3 \times -1 + 1$	– 2	(– 1, – 2)
0	$3 \times 0 + 1$	1	(0, 1)
1	$3 \times 1 + 1$	4	(1, 4)
2	$3 \times 2 + 1$	7	(2, 7)

The graph that is obtained when the function is plotted on a coordinate plane using the ordered pairs in the above table of values is shown below.



By observing the above graph, the following characteristics can be identified.

- It is a straight line graph.
- The straight line intersects the y - axis at (0, 1).
- The straight line makes an acute angle counterclockwise with the positive direction of the x- axis. The value of m of this line is + 3. This means that when the variable x increases by 1 unit, the variable y increases by 3 units.
- The value representing c in the equation y = $3x + 1$ is + 1. The y cordinate of the point at which the straight line intersects the y - axis is also 1. These two values are equal.

The distance, from the origin to the point where the straight line intersects the y - axis is known as the intercept. The intercept of this line is + 1.

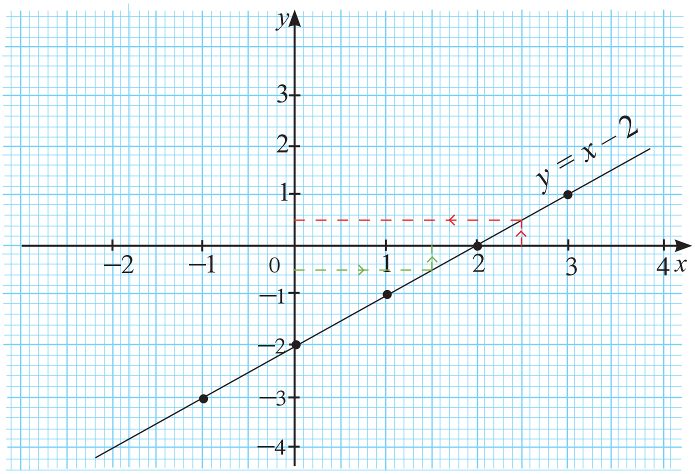
Accordingly, the gradient of the graph of a function of the form $y = mx + c$ is m and the intercept is c.

Example 1

Prepare a suitable table of values and draw the graph of the function $y = x - 2$. Using the graph, find the following.

i. The intercept.
ii. The value of y when $x = 2.5$
iii. The value of x when $y = -\frac{1}{2}$

x	-1	0	1	2	3
$y = x - 2$	-3	-2	-1	0	1



i. Intercept $(c) = -2$.

ii. $y = \frac{1}{2}$ when $x = 2.5$.

iii. $x = 1\frac{1}{2}$ when $y = -\frac{1}{2}$.

Example 2

Without drawing the graph, write the gradient and the intercept of the graph of each of the functions given by the following equations.

i. $y = -2x + 5$
ii. $y + 3x = -2$

i. $y = -2x + 5$ is of the form $y = mx + c$.
Accordingly, the gradient $(m) = -2$
The intercept $(c) = 5$

ii. Let us first write $y + 3x = -2$ in the form $y = mx + c$.
That is, $y = -3x - 2$.
Accordingly, the gradient $(m) = -3$
The intercept $(c) = -2$

Example 3

Prepare suitable tables for the functions $y = 2x$, $y = 2x + 1$ and $y = 2x - 3$, and draw their graphs on the same coordinate plane.

i. Write the gradient and intercept of each graph by observing the equation.
ii. Write a special feature that you can observe in the graphs.

$y = 2x$

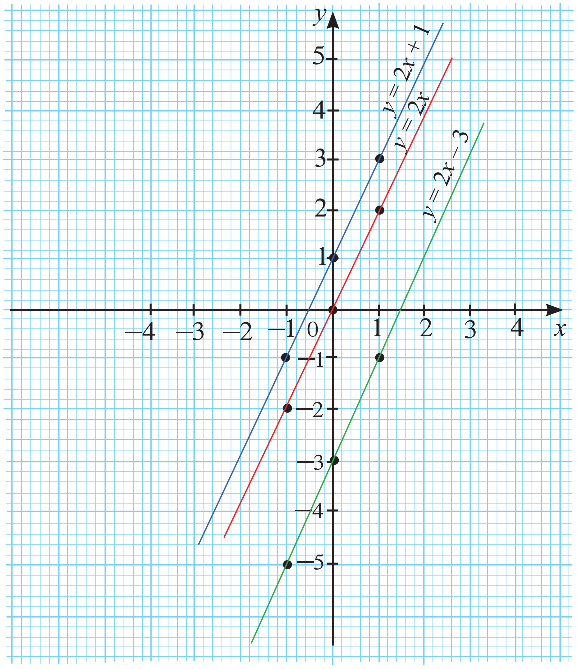
x	–1	0	1
y	–2	0	2

$y = 2x + 1$

x	–1	0	1
y	–1	1	3

$y = 2x - 3$

x	–1	0	1
y	–5	–3	–1



$y = 2x$
gradient = 2; intercept = 0

$y = 2x + 1$
gradient = 2; intercept = + 1

$y = 2x - 3$
gradient = 2; intercept = –3

By observing the equations of the graphs it is clear that the gradients of the three graphs are the same.By observing the graphs it can be seen that the lines are all parallel to each other. Therefore, it is clear that if the gradients of two or more linear functions are equal to each other, then their graphs will be parallel straight lines.

Graphs of functions given by equations of the form $ax + by = c$

Now let us consider the graphs of functions given by equations of the form $ax + by = c$. It is easy to prepare the table of values by writing this equation in the form $y = mx + c$.

Consider the following example.

Example 1

Prepare a suitable table of values and plot the graph of the function given by the equation $3x + 2y = 6$.

Using the graph that is drawn,

i. write the coordinates of the points at which it intersect the main axes.
ii. writethe gradient and intercept.

First let us change this equation to the form $y = mx + c$

$$3x + 2y = 6$$

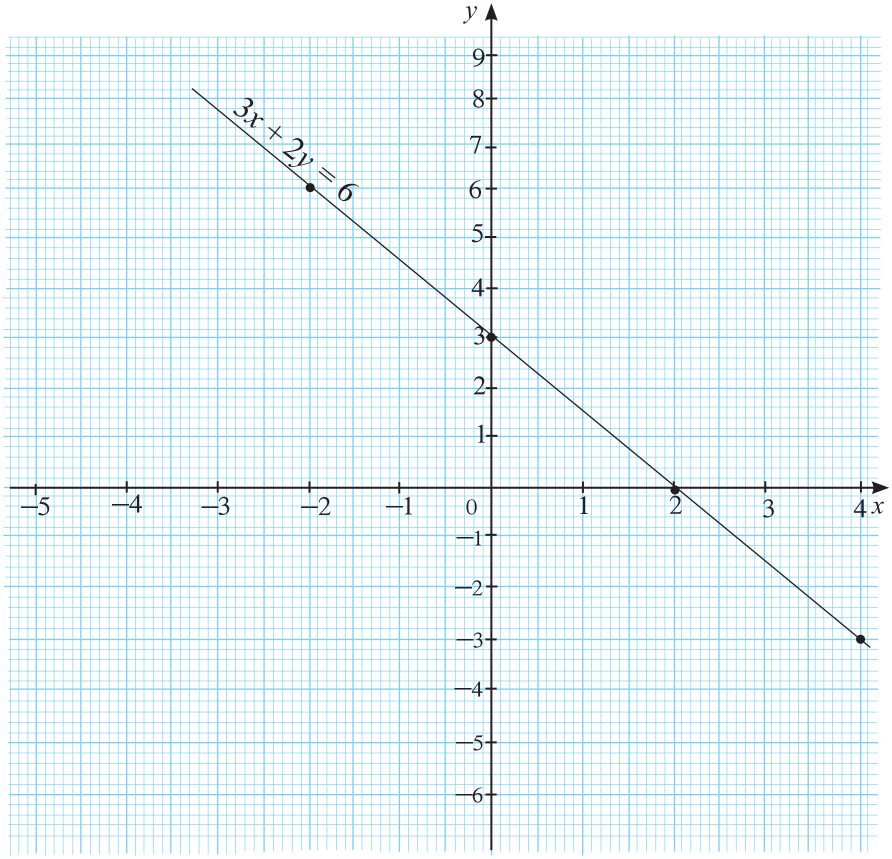
$$2y = -3x + 6$$

$$y = -\frac{3}{2}x + 3.$$

Let us obtain the coordinates of points that lie on the graph of this function from the following table of values and use them to draw the graph.

x	$-\frac{3}{2}x+3$	y
-2	$-\frac{3}{2}\times -2+3$	6
0	$-\frac{3}{2}\times 0+3$	3
2	$-\frac{3}{2}\times 2+3$	0
4	$-\frac{3}{2}\times 4+3$	-3

(– 2, 6) (0,3) (2, 0)(2, 0)(4, – 3)



i. The graph intersects the y - axis at (0, 3) and the x- axis at (2, 0).

ii. Gradient $(m) = -\frac{3}{2}$, intercept $(c) = 3$

Note:

- Observe that the graph of $3x + 2y = 6$ intersects the y axis at the point $(0, 3)$ and that the y coordinate of that point is equal to the coefficient of x in the equation $3x + 2y = 6$.
- Observe that the graph of $3x + 2y = 6$ intersects the x axis at the point $(2, 0)$ and the x coordinate of that point is equal to the coefficient of y in the equation $3x + 2y = 6$.
- Note that the graph of $3x + 2y = 6$ can be plottted by joining the points $(0, 3)$ and $(2, 0)$, without preparing a table of values.

Exercise 20.3

1. For each of the functions given by the equations in parts (a) and (b), write the gradient and intercept without drawing the corresponding graphs and write whether the graph makes an acute or obtuse angle counterclockwise with the positive direction of the x - axis.

(a) i. $y = x + 3$ **ii.** $y = -x + 4$ **iii.** $y = \frac{2}{3}x - 2$ **iv.** $y = 4 + \frac{1}{2}x$

(b) i. $2y = 3x - 2$ **ii.** $4y + 1 = 4x$ **iii.** $\frac{2}{3}x + 2y = 6$

2. For each of the functions given below, find the coordinates of the points at which the graph of the function intersects the two axes and plot the graph of each function using these two points.

(a) i. $y = 2x + 3$ **ii.** $y = \frac{1}{2}x + 2$

(b) i. $2x - 3y = 6$ **ii.** $-2x + 4y + 2 = 0$

3. Using the information given below, write the equation of each straight line.

Gradients (m)	Intercept (c)	Equation of the function
i. $+2$	-5	$y = 2x - 5$
ii. -3	$+4$	
iii. $-\frac{1}{2}$	-3	
iv. $\frac{3}{2}$	$+1$	
v. 1	0	

4. An incomplete table of values prepared to draw the graph of the function $y = -3x - 2$ is given below.

x	-2	-1	0	1	2
y	______	______	-2	______	-8

i. Fill in the blanks.
ii. Draw the graph of the above function.
iii. Draw the straight line given by $y = x$ on the same coordinate plane and write the coordinates of the point of intersection of the two lines.

5. By selecting suitable values for x, construct a table of values and draw the graphs of the following functions on the same coordinate plane.

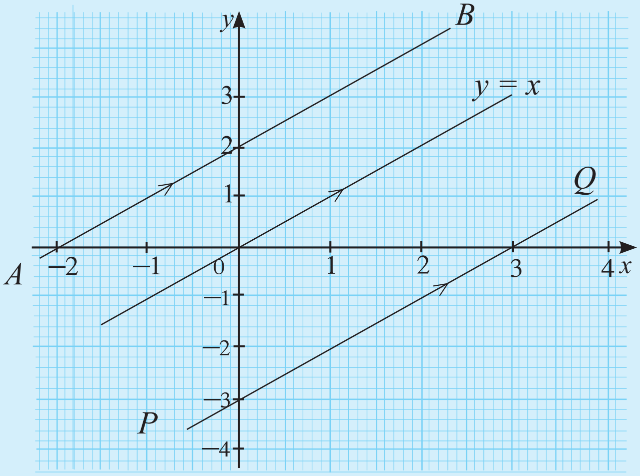
i. $y = x$ **ii.** $y = -2x + 2$ **iii.** $y = \frac{1}{2}x + 1$ **iv.** $y = -\frac{1}{2}x - 3$

6. Draw the graphs of the functions given by the following equations for the values of x from -4 to $+4$.

a. $-3x + 2y = 6$ and $3x + 2y = -6$

b. $y + 2x = 4$ and $-2x + y = -4$

7. Write the equations of the straight lines AB and PQ in the following figure.



Miscellaneous Exercise

1. For the statements given below, mark a “√” in front of the correct statements and a “×” in front of the incorrect statements.

i. For all m, the graph of a function of the form $y = mx + c$ is a straight line which is not parallel to the main axes. (............)

ii. For a function of the form $y = mx + c$, the value of m determines the direction of the straight line graph and the value of c determines the point where the graph intersects the y - axis. (............)

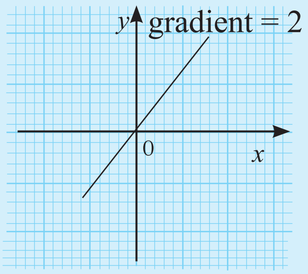
iii. It is not necessary for c to be zero, for the graph of a function of the form $y = mx + c$ to pass through the origin, (............)

iv. The graphs of the functions given by the equations $y_1 = m_1x + c_1$ and $y_2 = m_2x + c_2$ will be parallel when $m_1 = m_2$. (............)

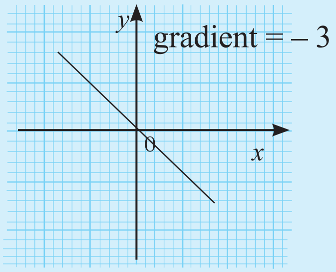
v. A straight line given by $y = mx + c$, intersects the y-axis above the x-axis only when $m > 0$, and $c > 0$. (............)

2. Write the equations of the functions of the graphs sketched below.

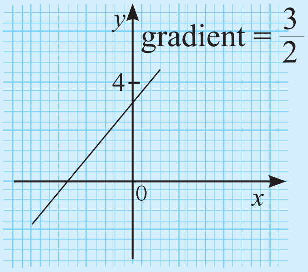
i.



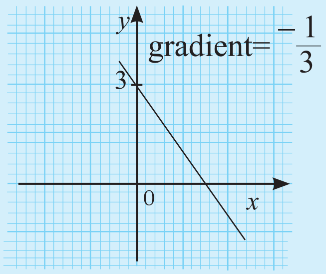
ii.



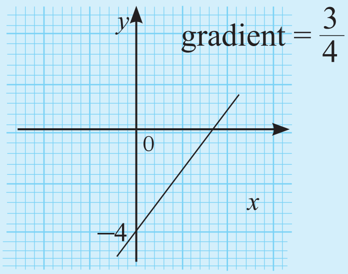
iii.



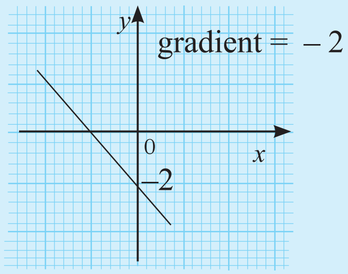
iv.



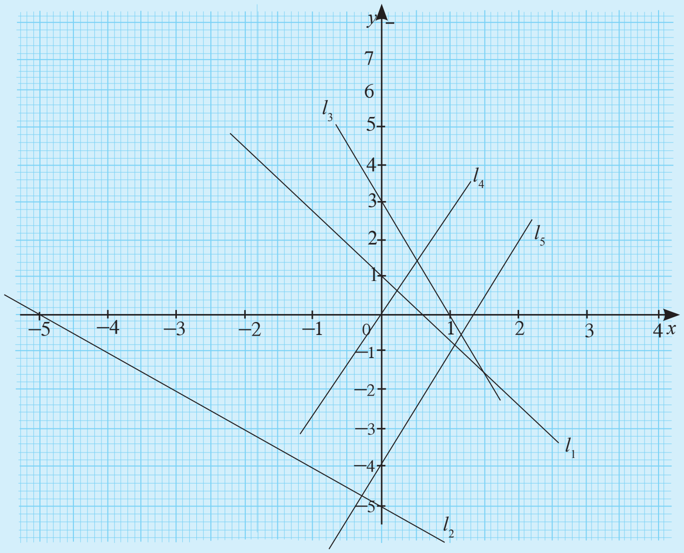
v.



vi.



3. Select and write the graph corresponding to each of the given functions.



Function

i. $y = 3x - 4$
ii. $y = - 2x + 1$
iii. $y = \ - x - 5$
iv. $y = - 3x + 3$
v. $y = + 3x$

4. The gradient of the straight line given by $4x + py = 10$ is $-\dfrac{4}{3}$.

i. Find the value of p. **ii.** Write the intercept.
iii. Write the equation of the straight line with gradient -2 which passes through the point at which the above given straight line intersects the y - axis.

Summary

- The gradient of the graph of a function of the form $y = mx + c$ is m and the intercept is c.

- If the gradients of two or more linear functions are equal to each other, then their graphs will be parallel straight lines.

Revision Exercise – Second Term
Part I

1. The price of a dozen books of a certain type is Rs 240. Find the maximum number of books that can be bought for Rs 150.

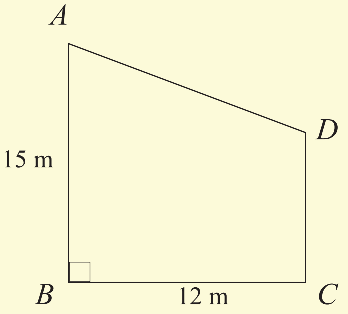
2. The price of a certain item is Rs 85000. If a discount of 20% is given when an outright purchase is made, using calculator find the amount that a customer has to pay when purchasing the item outright.

3. Simplify $\dfrac{(x^{-3})^0}{(2x^{-1})^2}$.

4. **i.** Round off 12.673 to the nearest second decimal place.
ii. Round off 4873 to the nearest hundred.

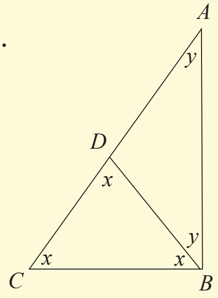
5. **i.** Write 5.62×10^{-3} in general form.
ii. Write 348 005 in scientific notation.

6. *ABCD* is the side view of a vertical wall of a house. If it is required to fix a bulb at an equal distance from *A* and *D* and 10 m from *B*, indicate its position by a rough sketch.



7. Solve $5\{3(x + 1) - 2(x - 1)\} = 10$

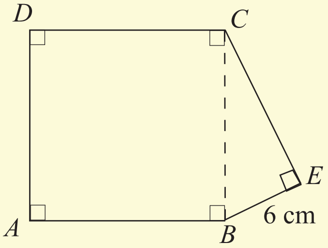
8. Find the values of x and y using the data in the given diagram.



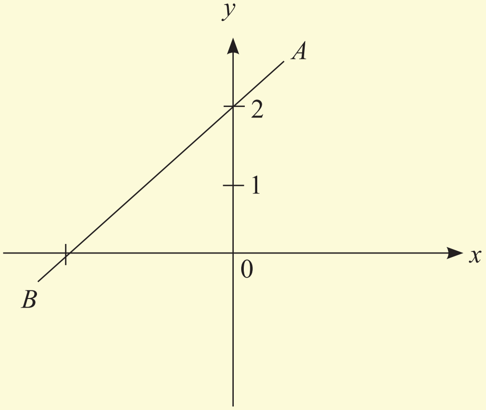
9. Make r the subject of the formula $V = I\,(R + r)$.

10. A square of side length 11 cm is made from a thin wire. Find the diameter of the largest circular bangle that can be made using this wire. (Use $\frac{22}{7}$ for the value π).

11. The area of the square $ABCD$ is 100 cm^2. If $BE = 6$ cm, find the perimeter of the figure.

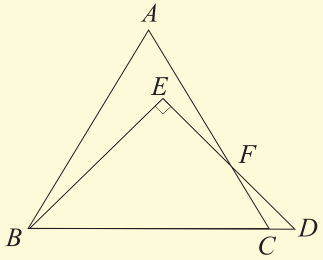


12. The gradient of the straight line AB is 3. Which of the following points are located on AB?
$(1, -5)$, $(-1, -1)$, $(\frac{1}{3}, -3)$, $(-\frac{1}{3}, 1)$



13. A group of Sri Lankans employed in a foreign country sent 25000 US dollars as aid for those affected by floods. How much is this amount in Sri Lankan rupees? (Assume that 1US dollar =150 Sri Lankan rupees)

14. ABC is an equilateral triangle. If $E\hat{F}A = 20^{\circ}$, find the magnitude of $A\hat{B}E$.



Part II

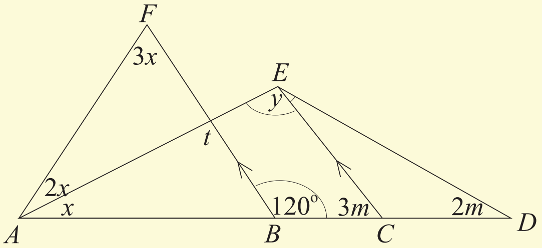
1.

x	-2	-1	0	1	2
y	-5	....	1	....	7

An incomplete table of values prepared to draw the graph of a function of the form $y = 2x - 1$ is given above.

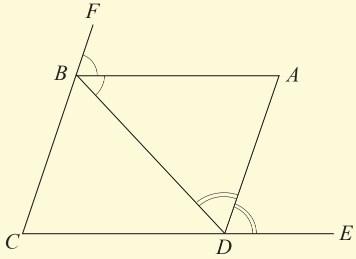
i. Fill the blanks in the table.
ii. Draw the graph of the above function on a suitable coordinate plane.
iii. If the point $(-5, k)$ is located on the above line, find the value of k.
iv. Write the equation of the straight line which is parallel to the line drawn in (ii) above and which passes through the point (0, 2).

2. (a) The straight lines BF and CE are parallel. Using the data given in the figure, find the following.



i. The values of x, t, y and m.
ii. The magnitude of $A\hat{E}D$.

(b) In the following figure, the bisectors of the angles $D\hat{B}F$ and $B\hat{D}E$ intersect at A.



i. Express the magnitude of $A\hat{B}D$ in terms of the magnitudes of $B\hat{D}C$ and $D\hat{C}B$.

ii. Express the magnitude of $A\hat{D}B$ in terms of the magnitudes of $D\hat{C}B$ and $C\hat{B}D$.

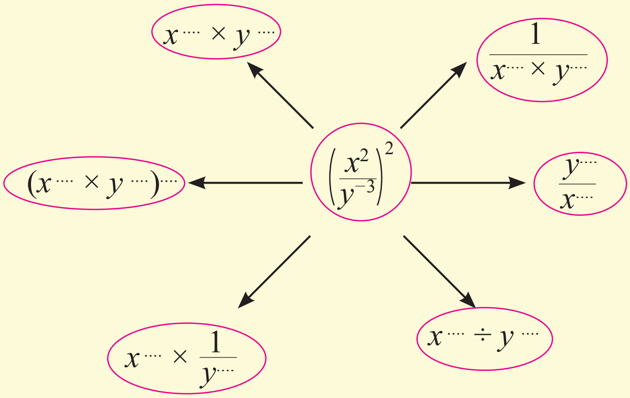
iii. Using the results of (I) and (II) above, show that $B\hat{A}D = 90^{\circ} - \dfrac{B\hat{C}D}{2}$.

3. (a) Simplify the expressions given below and write the answers in terms of positive indices.

(i) $$\frac{(a^{-3})^2\times(b^{\frac{1}{2}})^8}{(a^2\times b^3)^{-2}}$$

(ii) $$\frac{x^3\times(2y)^2\times t^3}{(2y^0)^3\times x^{-2}\times(t^{-\frac{1}{2}})^2}$$

(b) Fill in the blanks.



4. i. Draw a straight line segment AB such that $AB = 8$ cm. Mark the point C such that $B\hat{A}C = 60^\circ$ and $AC = 5$ cm.

ii. Draw the locus of points which are 2 cm from AB and located on the same side of AB as C.

iii. Mark the point P which is located on the locus drawn in (ii) above and is equidistant from AC and AB.

iv. Name the two points on AB which are 3 cm from P as Q_1 and Q_2. Measure and write the distance between Q_1 *and* Q_2.

5. Write the order in which you need to press the keys of a calculator to simplify the following expressions and obtain their values using a calculator.

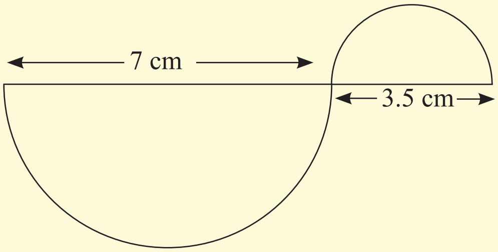
(i) $\frac{3.2\times5.83}{4.72}$ (ii) $\frac{2.5^2\times8.3}{4.7}$ (iii) $520\times20\%$ (iv) $\sqrt{\frac{20\times9}{5}}$

6. Express the distance to each planet from Earth in scientific notation.

i. The distance from Earth to planet A is 427 000 000 km.

ii. The distance from Earth to planet B is 497 000 000 km.

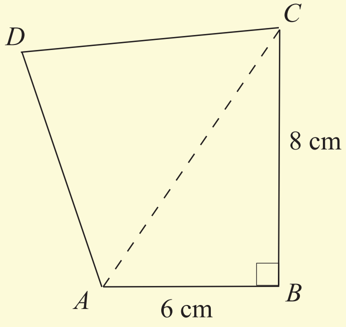
7. (a) The shape shown below is made using a thin metal wire. (Use $\frac{22}{7}$ for the value π).



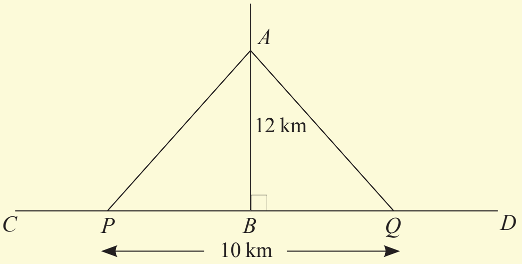
i. Find the total length of metal wire used to make this structure.

ii. If the price of 1m of this metal wire is Rs 120, find the price of the metal wire required for this structure.

(b) If the perimeter of the equilateral triangle *ADC* is 30 cm, find the perimeter of the figure.



8. As shown in the figure, *AB* and *CD* are two straight roads which are perpendicular to each other. *AP* and *AQ* too are two straight roads located as shown in the figure. A factory located at *A* produces two types of items. To store them, the warehouses *P* and *Q* located on CD at an equal distance from *B* are used. The distance between *P* and *Q* is 10 km. Giving reasons, explain which roads you would select to transport the items from *A* to *P* and *Q* using the same lorry so that the transport cost is the least.



9. (a) Make a the subject of the $A = \frac{h}{2}(a + b)$ formula. Find the value of a when $A = 70$, $h = 10$, $b = 8$.

(b) Solve.

(i) $2m + 3n = 6$
$2m - 7n = -14$

(c) Solve.

i. $2x + 3\{2(x + 2) + 3(x - 4)\} = 10$

ii. $\frac{2(x + 1)}{3} - 5 = \frac{x - 1}{3}$

iii. $3\left[1 + \frac{(2x - 1)}{3}\right] = 2(3 - x)$

10. **i.** If the price of a dozen eggs is Rs 186, find the price of 25 eggs.

ii. The price of 1 litre of petrol is Rs 117. A certain motorbike requires 3 litres of petrol to travel 180 km. What is the minimum amount he needs to spend on petrol to travel 330 km?

iii. A son employed in a foreign country sends 5000 Sterling Pound to his parents. What is the value of this amount in Sri Lankan rupees?
(Assume that 1 Sterling Pound = 190 Sri Lankan rupees).

Glossary

A

Algebraic form	වීජීය ආකාරය	அட்சரகணித வடிவம்
Axioms	ප්‍රත්‍යක්ෂ	வெளிப்படை உண்மைகள்

B

Bisector	සමච්ඡේදකය	இருகூறாக்கி

C

Capacity	ධාරිතාව	கொள்ளளவு
Circle	වෘත්තය	வட்டம்
Circumference	පරිධිය	பரிதி
Construction	නිර්මාණය	அமைப்பு
Constant distance	නියත දුර	மாறாத் தூரம்
Cube	ඝනකය	சதுரமுகி

D

Diameter	විෂ්කම්භය	விட்டம்
Direct Proportion	අනුලෝම සමානුපාතය	நேர்விகிதசம
Division	බෙදීම	வகுதத்ல்

E

Equal distance	සමාන දුර	சம தூரம்

F

Fixed point	අචල ලක්ෂ්‍යය	நிலையான புள்ளி
Foreign currency	විදේශ මුදල්	வெளிநாட்டு நாணயம்
Formula	සූත්‍රය	சூத்திரம்
Function	ශ්‍රිතය	சார்பு

G

Gradient	අනුක්‍රමණය	படித்திறன்
Graph	ප්‍රස්තාරය	வரைபு

H

Hypotenuse	කර්ණය	செம்பக்கம்

I

Index	දර්ශකය	சுட்டிகள்
Intercept	අන්තඃඛණ්ඩය	வெட்டுத்துண்டு
Interior angles	අභ්‍යන්තර කෝණ	அகக்கோணங்கள்
Intersection	ඡේදනය	இடைவெட்டு தல

K

Key	යතුර	சாவி
Key board	යතුරු පුවරුව	சாவிப்பலகை

L

Locus	පථය	ஒழுகக்

M

Multiplication	ගුණ කිරීම	பெருக்கல்

P

Parallel	සමාන්තර	சமாந்தரம்
Parallel lines	සමාන්තර රේඛා	சமாந்தரக்கோடுகள்
Perpendicular	ලම්බය	செங்குத்து
Perpendicular bisector	ලම්බ සමච්ඡේදකය	இருசமவெட்டிச் செங்குத்து
Power	බලය	வலு
Proportion	සමානුපාතය	விகிதசமன்
Pythagorus Connection	පයිතගරස් සම්බන්ධය	பைதகரஸ் தொடர்பு

Q

Quantity	රාශිය	கணியம்

R

Radius	අරය	ஆரை
Right angle	සෘජුකෝණය	செங்கோணம்
Right angled triangle	සෘජුකෝණික ත්‍රිකෝණය	செங்கோண முக்கோணி
Rules of indices	දර්ශක නීති	சுட்டி விதிகள்

S

Scientific notation	විද්‍යාත්මක අංකනය	விஞ்ஞான முறைக் குறிப்பீடு
Simple equations	සරල සමීකරණ	எளிய சமன்பாடுகள்
Simultaneous equations	සමගාමී සමීකරණ	ஒருங்கமை சமன்பாடுகள்
Straight line	සරල රේඛාව	நேர்கோடு
Subject	උක්තය	எழுவாய்
Substitution	ආදේශය	பிரதியிடல்

T

Theorem	ප්‍රමේයය	தேற்றம்
Triangle	ත්‍රිකෝණය	முக்கோணம்

U

Unknown	අඥාතය	தெரியாக்கணியம்

V

Verify	සත්‍යාපනය	வாய்ப்புப்பார்த்தல்

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